VIDYA PRASARAK MANDAL'S

B.N. BANDODKAR COLLEGE OF SCIENCE (AUTONOMOUS), THANE

NAAC Re-accredited 'A' Grade, College with Potential for Excellence (CPE),
Best College Award, IS/ISO 21001:2018 Certified (BIS), BEQET Special Citation Award,
Recipient of RBNQA Commendation Certificate from IMC,
Recipient of Jagar Janivancha Awards of Govt. of Maharashtra,
Supported Under - FIST 'O' Level (Min. of Sci. & Tech.) & DBT Star College Scheme.

Proceedings of the National Conference

Applied Mathematics
&
Statistics

13th & 14th January , 2023

In association with
University of Mumbai
&

Organized by

Department of Mathematics
&
Department of Statistics

Edited by :
Dr. Moses Kolet, Mrs. Minal Wankhede, Ms. Sumaiya Shaikh

Proceedings of National Conference

Applied Mathematics and Statistics

13th - 14th January, 2023

Jointly

Organized by

Department of Mathematics

&

Department of Statistics

Vidya Prasarak Mandal's

B. N. Bandodkar College of Science (Autonomous), Thane

NAAC Reaccredited 'A' Grade, College with Potential for Excellence (CPE), Best College Award,
Recipient of RBNQA Commendation Certificate from IMC,
Recipient of JagarJanivancha Awards of Govt. of Maharashtra, Supported Under Fist O Level
(Min. of Sci. & Tech.) & DBT Star College Scheme, BEQET Special Citation Award,
IS/ISO 21001 : 2018, BIS

In Collaboration with

University of Mumbai

Supported by

State Bank of India

Dr. V. V. Bedekar

Chief Patron

Dr. Moses Kolet

Principal, Convener

Mrs. Minal Wankhede and Ms. Sumaiya Shaikh

Organizing Secretaries

Patron

Dr. V. V. Bedekar

Convener

Capt. Dr. Moses Kolet

Organizing Secretary

Mrs. Minal Wankhede   Ms. Sumaiya Shaikh

Organizing Committee

Mrs. Umalaxmi Patne	Ms. Priyanka Chavan	Ms. Saba Pangarkar
Ms. Tanzim Shaikh	Mr. Sachin Gondke	Ms. Priyanka Rajput
Dr. Urmila Kumawat	Dr. Kiran Pariya	Ms. Asba Ansari
Ms. Priya Singh	Mrs. Rupali Ahire	

Editorial Committee

Dr. Moses Kolet	Mrs. Minal Wankhede	Ms. Sumaiya Shaikh
Ms. Tanzim Shaikh	Ms. Priyanka Chavan	Mrs. Umalaxmi Patne

Review Committee

Dr. Manisha Acharya	Dr. Santosh P. Gite	Dr. Sujata Suvarnapatki
Dr. Akanksha Kashikar	Dr. Chandrashekhar Meshram	Dr. Manoj Mishra
	Dr. Sakharam Muley	

Web & Library Support

Mrs. K. H. Manjrekar   Mrs. P. V. Mohite
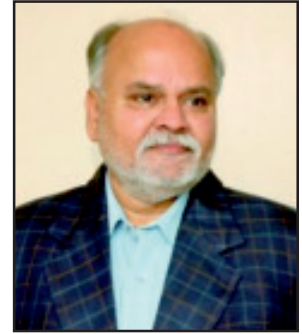
Digital Support

Mr. Abhijeet Kale   Mr. Tejas Jadhav

Support Staff

Mr. Dilip Ambhore   Mr. Mahesh Satle   Mr. Akshya Doltade

National Advisory Committee

- **Dr. Santosh P. Gite,** Professor, University of Mumbai, Mumbai.

- **Dr. Manisha Acharya,** Vice Principal, Maharshi Dayanand College, Mumbai.

- **Dr. Vijay M. Sarode,** Principal, G. M. Vedak College of Science, Raigad.

- **Dr. Dilip Patil,** Principal, N.K.T.T College of Commerce, Thane.

- **Dr. Sudhakar Agarkar,** Dean, VPM's Academy of International Education and Research, Thane.

- **Mr. Mandar Bhanushe**, Head, IDOL, University of Mumbai.

- **Dr. Sucheta Ketkar,** HOD & Vice Principal, Ramnarain Ruia Autonomous College of Science and Arts, Matunga, Mumbai.

- **Dr. Manoj Mishra,** Associate Professor, Guru Nanak Khalsa College of Arts, Science and Commerce, Matunga, Mumbai.

- **Dr. Dnyaneshwar Doke,** Principal, M.L. Dahanukar College, Vile Parle, Mumbai.

- **Dr. Sujata Suvarnapatki,** Associate Professor, Ramnarain Ruia Autonomous College of Science and Arts, Matunga, Mumbai.

- **Mrs. Pooja H. Ochaney,** Associate Professor (Retired), St. Xavier's College Autonomous, Mumbai.

- **Dr. Sanjeev Kumar Sinha,** Assistant Professor, Guru Nanak Khalsa College of Arts, Science and Commerce, Matunga, Mumbai.

- **Dr. Annapurna S.,** Associate Professor, St. Xavier's College Autonomous, Mumbai.

- **Dr. Shripad Garge,** Assistant Professor, IIT Bombay, Powai, Mumbai.

- **Dr. Sanket Tikre,** Assistant Professor, R. Jhunjhunwala College (Autonomous), Ghatkopar, Mumbai.

- **Dr. Akanksha S. Kashikar,** Assistant Professor, Savitribai Phule Pune University, Pune.

- **Dr. Chandrashekhar Meshram,** Assistant Professor, Chhindwara University, M.P.

- **Dr. P. N. Kamble,** Assistant Professor, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad.

Student Members

Ms. Shreya Nadkarni   Ms. Pradnya Barge   Mr. Annirudha Shelke

Ms. Neha Vishwakarma   Mr. Suraj Vishwakarma

Chairman's Message...

It gives me pleasure to address delegates of our National Conference on Applied Mathematics and Statistics.

Vidya Prasarak Mandal (VPM), Thane is a premier educational trust established in 1935 in the then small township of Thane and today catering to the needs of students through our many educational institutions.

Over the years, we have experienced adequate growth and we now run a variety of educational institutions, comprising two schools, a polytechnic, six colleges spreading across different faculties' viz. arts, science, commerce, law, management and engineering and 12 other institutions for overall growth of youth. Our higher educational campus 'Jnanadweepa' in Thane and college of engineering in Velneshwar, all have world class infrastructure.

VPM's group of institutions are serving the nation tirelessly, creating a strong and able workforce. Each institution on the campus is adapting to the changing requirements of education system and creating better knowledge ecosystem for learners.

B. N. Bandodkar College of Science (Autonomous) is committed to provide quality education in science. The fast pace with which the technologies are changing, keeping learners updated with current happenings in their Chosen fields of utmost importance. Anticipating this, hosting of National / International conferences annually has been included in our academic calendar. B. N. Bandodkar College of Science (Autonomous) is organizing National / International conferences every year which are conducted by various departments.

This year National Conference on 'Applied Mathematics and Statistics' is hosted by our Department of Mathematics and Department of Statistics. This is the opportunity and platform for professionals, academicians, researchers and students to come together and exchange their ideas.

The conference is intended for mathematicians and statisticians in a broad sense and aims to cover new developments and advances related to the applications of Mathematics and Statistics. I am sure this conference would bring together students, young amateur scientists, learners of different faculties and stalwarts in these as well as allied fields. We wish to create a rare and important connect across disciplines.

I wish the organizing departments and the college 'All the Best' for success of the conference.

Dr. Vijay Bedekar
Chairman,
Vidya Prasarak Mandal, Thane.

Convener's Address…

It gives me great pleasure to address this august gathering and present the proceedings of our National Conference on 'Applied Mathematics and Statistics' NCAMS-2023.The core subjects of Mathematics and Statistics play major roles in every field and have multi-faceted applications which are on the rise owing to digitalization all round us.

Applied Mathematics and Statistics encompass a broad spectrum of Subjects and cut across disciplines, widely extending scopes of the subjects .There are applications in several key fields and also involving techniques to arrive at inferences in uncertain circumstances. State of the art computational techniques have further facilitated calculations nearly impossible to perform manually.

Our departments of Mathematics and Statistics have jointly organized this National Conference to create a platform for veterans, young scientists, researchers and students to come together and exchange their views and work.

I am thankful to our Vidya Prasarak Mandal for sanctioning this conference; as well as the organizing departments, advisory committee, organizing committee and all committees, working team, student volunteers and all who have been working tirelessly towards success of the event.

I extend my Best Wishes for healthy deliberations and positive outcomes of the National Conference.

Wish you success and fulfillment.

Dr. Moses J. Kolet
Principal,
B. N. Bandodkar College of Science (Autonomous)

Organizing Secretaries Address…

Statistics and Mathematics are the most popular subjects across all fields. Mathematical Science has emerged out in a wide range of useful and impactful techniques to quantify the effect of uncertainty to the problems for making reasoning facts and effective decisions. Statistics deals with every piece of information right from the planning till the interpretation of the result to the problems. Reasoning about the set of information is recognized as a vital skill in the modern era. Knowledge of mathematical science and its technique is virtually utilized in all scientific disciplines like business, humanities, manufacturing, physical and social sciences, all engineering disciplines, space technologies, and beyond. It is crucial to keep encouraging new ideas and highlighting significant breakthroughs within these key fields of research.

National Conference on 'Applied Mathematics and Statistics' is a peculiar event of its kind bringing together professionals from around the nation contributing various ideas and thoughts of diverse fields. Conference aims to provide a high-visibility, open-access publishing outlet for the applied mathematics and statistics communities and not only welcomes submissions from fundamental areas of study, but also encourages those that focus on the development of these principles to other related fields. This approach presents novel opportunities to foster interdisciplinary research within mathematical and statistical disciplines, and provides a platform to highlight the latest developments in these continually evolving fields.

Mrs. Minal Wankhede and Ms. Sumaiya Shaikh

Editor's Note...

This publication contains the proceedings of the National Conference on 'Applied Mathematics and Statistics' held at VPM's B. N. Bandodkar College of Science (Autonomous), Thane on January 13 - 14, 2023.

The B. N. Bandodkar College of Science has earned a reputation throughout the years for its efforts in organizing state and national scientific conferences to stimulate students' interest in pursuing research. We anticipate that this national conference will contribute to further horizon expansion.

The theme of the conference is Applied Mathematics and Statistics. The fields of applied mathematics and statistics focus on applying mathematical techniques and logic to real-world challenges that are of a scientific or decision-making nature. Examples of such fields include engineering, medicine, the physical and biological sciences, and the social sciences. Systems of equations are frequently used in applied mathematical and statistical modeling to describe and forecast the behavior of extensive real-world systems that evolve dynamically over time. In its broadest sense, statistics is the study of uncertainty. It uses concepts from probability theory, statistical inference, decision theory, and processes from probability theory to address concerns like how to measure and what to do in a given scenario. Therefore, one of the conference's goals is to spark young people's interest in the developments of computational techniques in the applied research field.

The conference aims to offer a forum for open discussion between academicians, scientists from research institutions, and all other stakeholders. These individuals are known contributors to the subject of Applied Mathematics and Statistics. Through invited talks, research paper presentations, and poster presentations, this scientific conference will promise to be an excellent opportunity to learn about the most recent advancements.

We have made a sincere attempt to gather the huge knowledge shared throughout the conference and the pre-conference workshops in this proceeding.

Editorial Team

Schedule of the National Conference

APPLIED MATHEMATICS AND STATISTICS

Day 1: 13th January, 2023

Venue : Thorale Bajirao Peshwe Sabagruha, VPM Campus

Time	Contents & Topic
8.30 a.m. – 9.30 a.m.	**Registration and Breakfast**
9.30 a.m. – 10.15 a.m.	Inauguration and Release of e-Proceedings
10.15 a.m. – 11.00 a.m.	**Special Guest Address : Mr. Basudev Banerjee** Overview on TQM and Its Application in Educational
	Institutions Interactive Session
11.00 a.m. – 11.45 a.m.	**Key Note Address : Dr. Asha Jindal** GGSTATSPLOT: Visualization with Statistical Analysis
	Interactive Session
11.45 a.m. – 11.50 a.m.	Vote of Thanks
11.50 a.m. – 12.00 p.m.	**Tea Break & displaying Poster Competition**
12.00 p.m. – 12.45 p.m.	**Invited Talk: Dr. Prakash N. Kamble** Coping Imprecision in Real Life Situations by Fuzzy Mathematics ***Chairperson:*** *Dr. Nithya Narayana*
	Interactive Session
12.45 p.m. – 1.30 p.m.	**Invited Talk: Dr. Sudhakar Agarkar** Mathematics for Socio-Economic Development ***Chairperson:*** *Dr. Nithya Narayana*
	Interactive Session
1.30 p.m. – 2.30 p.m.	**Lunch Break**
2. 30 p.m. – 3.15 p.m.	**Invited Talk: Dr. Annapurna Shankarnarayanan** An Overview of Multivariate Analysis ***Chairperson:*** *Dr. Asha Jindal*
	Interactive Session
3.15 p.m. – 4.05 p.m.	Oral Paper Presentation Part I
	Interactive Session
4.05 p.m. – 4.30 p.m.	**Tea Break**
4.30 p.m.- 5.30 p.m.	**Cultural Event**

Day 2: 14th January, 2023

Venue: Thorale Bajirao Peshwe Sabagruha, VPM Campus

Time	Contents & Topic
9.00 a.m. – 10.0 a.m.	**Breakfast**
10.00 a.m. – 10.45 a.m.	**Invited Talk: Dr. Nithya Narayana** Math Matters ***Chairperson:*** *Dr. Prakash N. Kamble*
	Interactive Session
10.45 a.m. – 11.30 a.m.	**Invited Talk: Dr. Chhaya Lande** Mathematical Modelling of Water Table Fluctuation in Various Geological Situation ***Chairperson:*** *Dr. Prakash N. Kamble*
	Interactive session
11.30 a.m. – 11.45 a.m.	**Tea Break**
11.45 a.m. – 12.30 p.m.	**Invited Talk: Dr. Anjum Ara** Use of Multivariate Multiple Linear Regression Model in Statistical Analysis of Data ***Chairperson:*** *Dr. Santosh Gite*
	Interactive Session
12.30 p.m. – 1:15 p.m.	**Invited Talk: Dr. Santosh Gite** Regression and Classification in Machine Learning: Challenges and Research Directions ***Chairperson:*** *Dr. Anjum Ara Ahmad*
	Interactive Session
1.15 p.m. – 2.30 p.m.	**Lunch Break**
2. 30 p.m. – 3.50 p.m.	Oral Paper Presentation Part II
3.50 p.m - 4.30 p.m	**Poster Presentation**
4.30 p.m. – 4.45 p.m.	**Tea break**
4.45 p.m. – 5.30 p.m.	**Valedictory Talk : Dr. Shripad M. Garge** Application of Division Algebras in Wireless Communication
	Interactive Session
5.30 p.m. – 6:00 p.m.	Vote of Thanks

Contents

XII

SPECIAL GUEST ADDRESS

KEY NOTE ADDRESS

&

INVITED TALKS

Special Guest address

Overview on TQM and Its Application in Educational Institutions

Mr. Basudev Banerjee

Abstract

Total Quality Management abbreviated as TQM is not a technique. It is an approach to improve flexibility, Efficiency and Effectiveness of an organization in its entirety. Its three basic elements are: * People who have to be given GRAPES (Growth, Recognition, Achievement, Participation, Esteem and Self-Actualization).

* Tools and Techniques such as SPC, Six Sigma, OR, BM, BPR etc. and

* Systems like ISO 9001 0n QMS, ISO 14001 on EMS, ISO45001 on SMS, NAAC, TS 16949 etc

TQM gradually got extended to TQOM (Total Quality of Management) which is nothing but World Class Management *WCM). In this context the Western World led by USA took to Six Sigma and Lean Six Sigma route and Eastern World led by Japan tool to 5S, TPM, Kaizen, Lean route to reach the summit of World Class Statue (WCS).

The revolution came in the area of "Quality". Technocrats defined it as (a) Adherence to laid down standards (b) Satisfying customers' specific needs (c) Dimensional requirement (d) Aesthetic requirement (e) Statutory requirement (f) Safety (g) Reliability & Maintainability (h) meeting BIS Standards etc. etc. All the above are "Product Control Philosophy" which is nothing but segregating Good from Bad. This is a postmortem analysis of the whole situation. You can satisfy customers with 20% rejection, 30& rework but what will happen to your own organization. Then the concept of :Process Control came up. This is a corrective philosophy. Then came the concept of Pre-Process Control which is control at Design stage leading to Prevention of non-conformities

Being in the field of Quality & Systems I define Quality as the gap between customers exact requirement and Manufacturers = economic process capability. The wider the gap is worse is the quality, the narrower the gap is better is meeting of minds.

Noting that variation is inevitable in nature and it is of two types chance or common causes and assignable or special causes we have to statistically control the later ones. Boundary lines between them come out of Cost-Benefit analysis.

As I mentioned earlier all forward looking organizations are vying for WCS (World Class Status) for this they are opting either for IMC-RBNQA (Ramkrishna Bajaj National Quality Award) administered by Indian Merchants' Chamber and/or DAP (Deming Application Prize). For either of them the organization has to draw up its V-M-V (Vision, Mission Values) document. They will be supported by Strategic plans and Policies leading to Objectives & Targets. Finally all the above arrest on sound Systems and Controls. The existing Systems &Controls are basically developed by UK (e.g. ISO Systems), USA (e.g. Six Sigma) and Japan (e.g. 5S, TPM, Lean).

IMC-RBNQA which started in 1997 with small, medium and large Business categories extended its wing to Education (both profit making and nonprofit making), Health Care (both profit making and nonprofit making), and Overseas and even for NGOs.

In Education category of IMC-RBNQA comprises six Enablers namely Leadership, Strategic Planning, Customer & Market focus, Measurement, Analysis and Knowledge Management, Workforce focus and Operations focus. They carry 550 points. Results in Education Category carry 450 points. They come under five headings as follows:

Stugemt Learning and Process outcomes (13), Customer focused outcomes (5), Workforce focused outcomes (9), Leadership and Governance outcomes (7) and Budgetary, Financial and Market outcomes (4)

Note: Figures inside brackets give number of parameters to be addressed.

RBNQA is a competitive Award whereas DAP is a noncompetitive one.

Keynote address

GGSTATSPLOT : Visualisation with Statistical Analysis

Dr. Asha Jindal

Professor and Head, Department of Statistics, K. C. College of Arts, Science and Commerce.

In Traditional teaching learning environment, Data visualization and statistical modelling are two different phases: visualization informs modelling, and modelling in its turn can suggest a different visualization method, and so on and so forth. GGSTATSPLOT is an extension of ggplot2 package for creating awesome plots having details of statistical tests. It's designed for making publication ready statistical plots. GGSTATSPLOT simply combine above mentioned two phases into one in the form of graphics with statistical details, which makes data exploration simpler and faster with a one line code creating information-rich plots.

Presently it supports statistical tests which includes parametric, nonparametric, robust, and Bayesian versions of t-test/ANOVA, correlation analyses, contingency table analysis, meta-analysis, and regression analyses.

Moreover, it produces plots for the supported analyses:

- violin plots (for comparisons between groups or conditions),

- pie charts (for categorical data),

- scatter plots (for correlations between two variables),

- correlation matrices (for correlations between multiple variables),

- histograms (for hypothesis about distributions), and

- Dot-and-whisker plots (for regression models).

Additionally, GGSTATSPLOT provides grouped_ versions of all functions that make it easy to repeat the same analysis for any grouping variable.

Mathematics for Socio-economic Development

Dr. Sudhakar C. Agarkar

VPM's Academy of International Education and Research, Thane, India

Mathematics is known as the Queen of all sciences. It has certainly enabled the enhancement of different branches of natural sciences. In addition, it has also offered the helping hand to boost the progress of social science disciplines like economics, sociology, education, etc. Apart from its role in academic sectors mathematics has played a great role in Socio-economic development of mankind as it can quantify social matters. Business management, all over the world, heavily depends on statistical techniques made available by mathematicians.

Soon after independence India embarked on five-year plans to realise its ambitious developmental strategy. These plans were both data driven and time bound. Wherever possible available data was made use of to work out the course of action. To collect fresh and relevant data a special organization named National Sample Survey (NSS) was set up by the Government of India. It has a mandate to collect and analyse information related to crucial social issues like education, health, infrastructure, etc. Based on these data the central government makes appropriate budgetary provision for different sectors. This mode of systematic work has resulted into a planned Socio-economic development of the country.

The literacy rate in India at the time of independence was below 20 percent. Through planned efforts it could be boosted to more than 80 percent in spite of the fact that the population over the years has trebled.

Similarity the per capita consumption of electricity was barely 16.3 units 1947. By constructing adequate number of thermal, hydroelectric and nuclear power stations and by designing proper distribution system the per capita consumption of electricity could be increased to 1208 in 2021. This developmental trend is certainly the result of a systematic plan based on statistical techniques used for data collection and analysis. The session will focus on the role of mathematics and statistics in bringing out notable social and economical changes in the country.

An Overview of Multivariate Analysis

Dr. Annapurna Shankar Narayanan,

PhD Associate Professor, Department of Statistics, Vice Principal (Arts), St. Xavier's College, Autonomous, Mumbai

Since the audience in this national conference comprise of students from the undergraduate three-year program in Statistics and have not been initiated into the realm of multivariate analysis, the following would give an understanding of the usefulness and objectives of multivariate studies. It will familiarize students with the importance of data arrangement and introduce them to a few techniques adopted for Multivariate Analysis (MVA), thus being the catalyst for further learnings.

MVA can be defined as the statistical study of data where multiple measurements are made on each experimental unit in order to understand the dependencies and interdependencies among them as well as to predict the patterns between them.

Hence the relationships among measurements as well as their structural form are important aspects of multivariate study as they aid in predicting future outcomes, improving efficiency, making beneficial decisions for organisations about processes and gaining new insights.

For example, to study the possible causes of hypertension, data needs to be collected on a range of variables such as age, diet, occupation, body weight, blood glucose, cholesterol, thyroid, family history and many more factors.

The multivariate methods that are used for investigation generally lend themselves to objectives such as data reduction, sorting and grouping into classes, understanding dependence among variables, predicting values of variables and construction and testing of hypothesis.

Since measurements are made on several variables and further analysed, the data is best organised and manipulated as a matrix of sample values and is then arranged and displayed in different manners such as graphs and tables.

In MVA, the joint analyses of multiple dependent variables are considered where the variables need to be organised using matrices and vectors. This representation simplifies notations and creates a format for expressing important formulae termed as the data matrix. The data matrix can comprise of rows as variables and the columns as observations. Such a data matrix $\underline{X}_{pXn}$ will be a 'p × n' matrix with n = the number of observed units and p = number of variables measured on each unit.

$$\underline{X}_{pXn} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n-1} & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n-1} & x_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{p1} & x_{p2} & \cdots & x_{pn-1} & x_{pn} \end{bmatrix}$$

In order to extract relevant information contained in large and bulky data sets, summary statistic can be calculated similar to those done in univariate analysis. The descriptive statistics measuring location, variation and linear association for multivariate study are of great use, the formal definitions of which are akin to univariate studies and hence need not be elaborated here.

Sample Mean Vector = $\overline{\underline{X}_{pX1}} = \begin{bmatrix} \overline{x}_1 \\ \overline{x}_2 \\ \vdots \\ \overline{x}_{p-1} \\ \overline{x}_p \end{bmatrix}$

Sample Variance-Covariance Matrix =

$$S_n = \begin{bmatrix} s_{11} & s_{12} & s_{13} & \cdots & s_{1p} \\ s_{21} & s_{22} & s_{23} & \cdots & s_{2p} \\ s_{31} & s_{32} & s_{33} & \cdots & s_{3p} \\ \vdots & \vdots & & \ddots & \vdots \\ s_{p1} & s_{p2} & s_{p3} & \cdots & s_{pp} \end{bmatrix}$$

Corresponding sample correlation matrix =

$$R = \begin{bmatrix} 1 & r_{12} & r_{13} & \cdots & r_{1p} \\ r_{21} & 1 & r_{23} & \cdots & r_{2p} \\ r_{31} & r_{32} & 1 & \cdots & r_{3p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{p1} & r_{p2} & r_{p3} & \cdots & 1 \end{bmatrix}$$

In similar manner the corresponding population structure can be stated with the vector $\underline{X}_{pX1}$, population

mean vector $\underline{\mu}_{pX1}$ and population variance-covariance matrix $\sum_{pXp}$.

Picturing multivariate data in a p dimensional space is not possible for the human mind which is limited to a maximum of three dimensions. A graphical representation of multivariate data can be created in two dimensions with box plots as the diagonal elements and scatter plots arranged as the off-diagonal elements of the covariance array. Such an arrangement could give information on symmetry, possible outliers, patterns and unusual observations. Similarly, a three-dimensional scatter plot can be formed if the core of the multivariate problem under study can be captured in three dimensions. The uses of sophisticated statistical software for data analysis and graphics have helped create visual aids to represent pictorially multidimensional data in two dimensions. Chernoff faces and Star plots are two such popular pictorial representations.

Chernoff faces is a data visualization technique that displays multivariate data effectively in the shape of a human face. The facial features such as eyes, ears, mouth, nose, eyebrows etc are mapped to multiple variables, with size, orientation, shape, colour, and placement potentially representing different attributes of a single observation. This interesting way to represent multivariate data can be used to detect similarities between different items but it is not the most efficient or the most accurate way to do so.

Another technique called star graphs, or radar charts consists of a sequence of equally spaced rays/spokes called radii emanating from the centre of a circle. Each spoke represents a variable and its length is proportional to the magnitude of the variable. A line connecting all the data values for each ray gives the plot a star-like shape and helps to identify outliers and similarities i.e. clusters of observations.

We know that many natural phenomena can be modelled using the normal distribution. Due to this, the multivariate normal distribution which is a generalisation of its univariate counterpart is useful in multivariate studies. The sampling distributions of many multivariate statistics are approximately normal due to the central limit theorem. Further many of the multivariate techniques assume the vector X comes from a multivariate normal distribution and hence the multivariate normal distribution can be considered as an important distribution.

We now look at some popular techniques of MVA. For a student to comprehend the various techniques in a more in-depth manner, the student will require to learn the initial aspects of multivariate study in detail.

Principal component analysis (PCA), is used to reduce the dimensionality of large data sets, by transforming a large set of variables into a smaller one while preserving most of the information available in the large set, thus making it easier to explore and visualize. The approach used is to standardise variables, use the covariance matrix elements, compute eigenvalues and eigenvectors and construct principal components that are linear combinations or mixtures of the initial variables. The combinations are created in such a way that the new variables (i.e. principal components) are uncorrelated and maximum information from the initial variables is compressed into the first component, then maximum remaining information in the second and so on. The first principal component accounts for the **largest possible variance** in the data set. The second principal component is calculated in the same way, with the condition that it is uncorrelated with (i.e., perpendicular to) the first principal component and that it accounts for the next highest variance. PCA has wide applications in image processing, customer profiling and various sectors like banking, healthcare, finance etc.

Factor Analysis is another multivariate technique that is applied to reduce large number of variables into a fewer number of factors based on covariance structures. The factor model is created such that variables with high correlation among themselves are grouped as a factor and the correlation of this group with any other group so created will have a relatively smaller correlation. The factors (latent or unobserved variables) create commonalities in some of the observed variables. The response may be similar when questions are posed about income, education, owning vehicle and occupation, which are all associated with the latent variable Socio-Economic Status of Individual. Similarly, variables such as home valuation, number of schools in the vicinity, proximity to shopping areas, accessibility to transit etc will have high correlations and can be considered as a factor Socio-Economic

Status of Locality. Factor analysis can be considered as an extension of PCA and in both, the dimension of the data is reduced. Factor analysis is applied in a multitude of fields such as psychology from where it originated, finance, marketing, health, intelligence, machine learning, ecology among others.

Another interesting technique is Discriminant Analysis which organizes observations into non-overlapping groups, based on measurements on one or more quantitative predictor variables. It is a tool for classification of new observational units into the groups or categories For example, a discriminant analysis could be conducted to identify if patients are at a high or low risk for stroke. This analysis might classify patients into high risk or low risk groups, based on personal features (e.g. weight, sugar levels, cholesterol level, body mass) and/or lifestyle behaviours (e.g. number of times outside food eaten per week, hours of exercise per week, number of cigarettes per day).

Thus, we understand that MVA is a very useful method to study relationships and evaluate patterns among large sets of data. However, it is complex and the process of designing the study and interpreting of results is a tedious one, but it gives valuable inputs and many benefits.

Math Matters

Dr. Nithya Sai Narayana

Associate Professor (Retired), N.E.S. Ratnam College, Bhandup, Mumbai

Mathematics is in general associated with numbers and equations and anything which looks for precision and prediction needs mathematics. Mathematical reasoning and logic are applied in the analysis of data, identification of patterns, modelling, and predicting values. Artificial Intelligence is a scientific area associated with data science where computers are made to mimic the problem-solving and decision-making capabilities of human beings. Mathematical concepts are used in face recognition, voice recognition, email spam filters, Chatbot, recommendations display, language translation, etc.

Real-life problems are modelled and solved using mathematical tools. Matrix theory, Eigenvalue, and Eigenvectors play major roles in prediction, data analysis, and also in graphs. There is a close relationship between the number of spanning trees and eigenvalues of the Laplacian matrix of the graph as revealed in the Matrix Tree theorem. The analysis of the eigenvalues of Laplacian reveals many properties of the graph including its connectivity,

Support Vector Machine is the concept used in data sciences and the mathematics behind this concept will help us to understand how optimization techniques are used to design models to perform predictions. Hyper planes, linear separability, topology, Lagrange multiplier methods, and quadratic programming are involved to arrive at appropriate models.

The fields that use strong mathematical concepts and tools keep growing day by day and mathematics has become an inseparable part of our life

Mathematical Modeling of Water Table Fluctuation in Various Geological Situations

Dr. Chhaya Lande

Assistant Professor, Applied Science Department, Symbiosis Institute of Technology, Pune

Water is one of the most important substances on earth for the survival of all plants and animals. If there was no water, there would be no life on earth. Our planet has an estimated 330 million cubic miles of water, most of which occurring as non-potable sea water (97%). Large volume of fresh water is frozen in glaciers and is essentially unavailable. Other reservoir such as streams, lakes etc. have local importance; however, they are much less significant on a global scale. Groundwater is one of the most important and safe resources of freshwater available to human being. Although, it makes up only 0.07% of earth's available potable water; yet, this relatively small volume is critically important because it represents 98% of fresh water readily available to human being. Thus the ground water represents the largest reservoir of fresh water. It is obvious that ground water, in terms of both economics and human welfare is more valuable. Consequently, its sound development, diligent conservation and consistent protection are important concerns of everyone. These concerns can be translated in to effective action only by increasing our knowledge of basic aspects of groundwater hydrology. Prior knowledge of aquifer's response to recharge and withdrawal is vital for planning and implementing the resource management related projects.

Mathematical models:

Mathematical models have emerged as important tools for prediction of transient as well as steady-state behavior of water table in unconfined aquifers under seepage and recharge conditions.

Most of the existing models are based on a restrictive assumption that the aquifer is underlain by a perfectly impervious and horizontal base. In reality aquifers are often connected to each other through their semi pervious base and mostly are in sloping nature.

Model: Analytical quantification of stream aquifer interaction in presence of thin vertical clogging layer.

An analytical solution is developed which can quantify the interaction of water in an archetypical stream-aquifer system under complex stratigraphic conditions. An unconfined aquifer overlaying sloping impervious bed interacts with a stream of varying water level in presence of thin sedimentary layer. A phreatic aquifer is being considered as shown in Figure 1. The aquifer is hydraulically connected with a fully penetrating stream whose water level is gradually varying from an initial value h_i to a final value h_f. As it often occurs in reality, a thin vertical clogging layer of conductivity k and thickness b is present between the stream and aquifer.
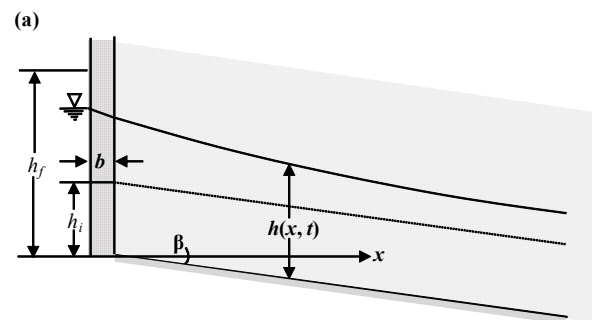
Figure 1: Schematic diagram of an unconfined semi-infinite aquifer overlying an impervious downward sloping base and interacting with a fully penetrating stream in presence of thin vertical sedimentary layer. Variations in water table are induced by rise.

The model is solved analytically and validate using numerical method. Considering hypothetical data simulation is done. The water head profile is studied. Figure 2 shows the water head profile for various sloping angle.
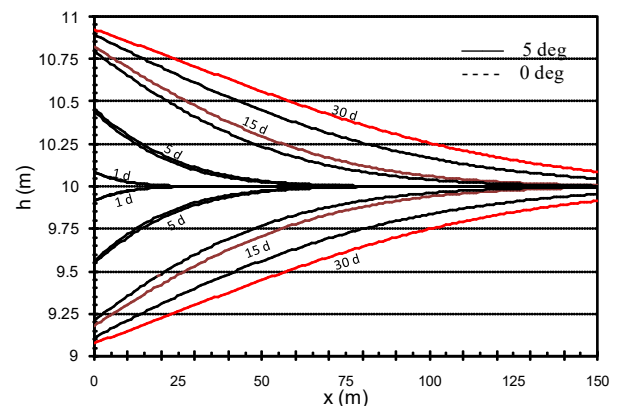
Figure 2: Water head altitude at a fixed point $x = 50$ m in aquifers with slope angle β = 5, 3, 0, -3, -5 deg due to gradual rise and decline in stream-stage (λ= 0.2 d^{-1}) and stream bank leakance $r = 10$ m

Conclusion :

In both gaining and loosing streams; recharging or discharging of sloping aquifer is mainly controlled by its base angle β.

On the other hand, occurrence of inflow or outflow at streambed-aquifer interface depends on the interrelationship between parameters β and λ.

References :

- Akylas E, Koussis AD (2007) Response of sloping unconfined aquifer to stage changes in adjacent stream. I. Theoretical analysis and derivation of system response functions. J Hydrol338: 85–95

- Bansal RK, Das SK (2009) Effects of bed slope on water head and flow rate at the interfaces between the stream and groundwater: analytical study. J HydrolEng14 (8): 832–83.

- Childs EC (1971): Drainage of groundwater resting on a sloping bed. Water Resour Res 7(5): 1256–1263

- Lande C, Bansal R.K., Warke A (2016), Simulation of 2-dimensional subsurface seepage flow in an anisotropic porous medium, Elsevier, Perspective in science, 8, 276-278.

- Lande C, Bansal R.K., Warke A (2020), Simulation of 2-dimensional subsurface seepage flow in Isolated Anisotropic Aquifer, Elsevier, Material today Proceedings,23, 329-337

Use of Multivariate Multiple Linear Regression Model in Statistical Analysis of Data

Dr (Mrs.) Anjum Ara Ahmad

Professor of Statistics, Rizvi College of Arts, Science & Commerce, Bandra West, Mumbai 400050

Abstract

The paper is based on how and when to use Multivariate Multiple Linear Regression Model in Statistical Analysis of Data. The paper will elaborate in detail the Multivariate Model with assumptions and the process how analysis can be done using the said Model. In order to estimate the effects in the multivariate multiple regression model, a statistical software IBM Statistical Product and Service Solutions originally called Statistical Software for Social Sciences SPSS (Version 19) has been used. The GLM option is used to analyse the data. GLM (general linear model) is a procedure which gives multivariate analysis of variance and covariance, as well as multiple regression analyses results in the same output. To explain the model, I have used by Ph.D. thesis data. The Model is used when there are more than one dependent variable and when the dependent variables are mostly correlated with each other. The Data used in the study is from Indian Census and the problem is to estimate the effect of Female fertility, Infant Mortality and Female Work Participation on Female Literacy Rates in Four States mainly Maharashtra, Kerala, Bihar & Jharkhand. Besides Female Literacy Rates, other variables which might affect the dependent variables are also considered in the Model.

Regression and Classification in Machine learning : Challenges and Research Directions

Dr. Santosh Gite

Professor ,Department of Statistics, University of Mumbai, Mumbai.

Abstract

We are in the age of the fourth industrial revolution era, in this digital era has a lot of information gathering each day such as business-related data, internet of things data, health industry related data, mobile data, social media data (Facebook, twitter, Instagram, etc.), cyber security data, e-commerce data etc. Usually, data is available in structured, semi- structured and unstructured format in real world. Knowledge of the machine learning is the key to analyze these data intelligently. Several machine learning algorithms are existed to analyze real world data such as supervised, unsupervised, semi-supervised and reinforcement learning. In this paper, different machine learning techniques particularly regression and classification techniques are explained and their applicability in various real-world domains such as agricultural, healthcare, e-commerce and social media etc. Also, I have highlighted potential research direction and challenges in given area.

Key words: *Machine learning, Data science, prediction etc.*

Valedictory Talk

Applications of Division Algebras in Wireless Communications

Dr. Shripad M. Garge

Department of Mathematics, IIT Bombay
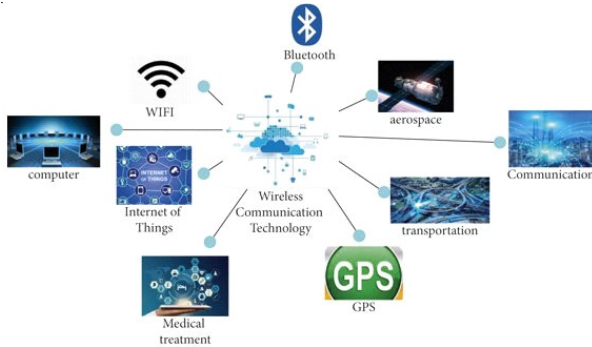
Multiple Input Multiple Output
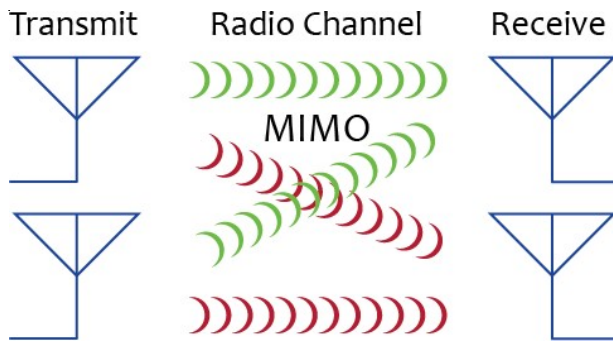
Basic Setup

Each radio wave is encoded by a complex number $re^{i\varphi}$. These numbers are sent from t transmit antennas and the encoded signals are then received by r receive antennas. The presence of obstacles in the environment such as buildings causes attenuation of the signals; in addition, the signals are reflected several times and interfere with one another. The combined degradation of the signals is commonly referred to as "fading", and achieving reliable communication in the presence of fading has been the most challenging aspect of wireless communication!

The received and transmitted signals are modeled by the relation

$$Y_{r\times 1} = \theta H_{r\times t} X_{t\times 1} + W_{r\times 1}$$

X is a $t \times 1$ vector of transmitted signals, Y is an $r \times 1$ vector of received signals, W is an $r \times 1$ vector of additive noise, H is an $r \times t$ matrix that models the fading, and θ is a real number chosen to normalize the transmitted signals so as to fit the available power.

The transmission typically occurs in blocks of length n: each antenna transmits n times, and the receiver waits to receive all n transmission before processing them. A common engineering model is to assume that $r = t = n$, so the equation above is accordingly modified to read

$$Y_{n\times n} = \theta H_{n\times n} X_{n\times n} + W_{n\times n}$$

Thus, the i-th column of Y, X, and W represent (respectively) the received vectors, the transmitted information, and the additive noise from the i-th transmission.

Coding:

A subset $S \subseteq \mathbb{C}^\times$, known as the *signal set* is selected as the alphabet.

A k-tuple $(s_1, s_2, \ldots, s_k)$, $s_i \in S$, comprises the message that the transmitter wishes to convey to the receiver.

A *space-time code* is then a one-to-one map $X : S^k \to M_n(\mathbb{C})$. It is typically assumed that the map X is " linear in S^k ", that is, it is the restriction to S^k of a group homomorphism $\langle S \rangle^k \to M_n(\mathbb{C})$, where $\langle S \rangle$ is the additive subgroup of $\mathbb{C}$ generated by S.

Probability of errors

Reliability of communication is commonly measured by the probability P_e of incorrectly decoding the transmitted message at the receiver. The pairwise error probability $P_e(i,j)$ is the probability that message i is transmitted and message j is decoded. Performance analysis of MIMO communication systems typically focuses on the pairwise error probability as it is easier to estimate and also because the error probability P_e can be upper and lower bounded in terms of the pairwise error probability.

Rank criteria

It was shown (2000) that in order to keep the pairwise error probability low, the space time code must meet the criteria below:

For $(s_1, s_2, \ldots, s_k) \neq (s_1, s_2, \ldots, s_k)$, $X(s_1, s_2, \ldots, s_k)$ -$X(s^2 1, s^2 2, \ldots, s^2 k)$ must have full rank n.

That is, it must be invertible.

Alamouti

The earliest space-time code, for two antennas, was given by an engineer, Alamouti: given an arbitrary signal set S, he chose

$X\colon S^2 \to M_2(\mathbb{C})$ to be $X(s_1, s_2) = \begin{pmatrix} s_1 & -\overline{s_2} \\ s_2 & \overline{s_1} \end{pmatrix}$

(where $\overline{s_i}$ stands for complex conjugation.) It is easy to see that the rank criterion is immediately met.

After Alamouti

Alamouti's code led to a furious search among engineers and coding theorists for generalizations for the higher number of antennas. Much of the early work focused on some combinatorial variations of Alamouti's code. Alamouti's code led to a furious search among engineers and coding theorists for generalizations for the higher number of antennas. Much of the early work focused on some combinatorial variations of Alamouti's code. It took the genius of **an Indian mathematician** to solve this problem in the general case.

Rajan and Sethuraman

In 2001, Sundar Rajan, a professor of communication engineering at IISc introduced the problem of designing matrices $X(s_1, \ldots, s_k)$ satisfying the rank criterion to Bharath Sethuraman, a mathematician at the CSUN, USA. Given his algebraic background, Bharath could recognize easily that matrices arising from embeddings of fields and division algebras can be utilized to solve this problem.

Division algebra

Let V be a vector space over a field. If you have a multiplication of vectors defined on this vector space: $m : V \times V \to V$ such that every nonzero vector has a (multiplicative) inverse then V is said to be a division algebra.

Examples of division algebras include

* C as a division algebra over R,

* any field over itself,

* Q[ɩ] over Q,

* R or Q(x) over Q, etc.

Rajan-Sethuraman codes

The major problem Sethuraman faced was that there are not $\in$ too many division algebras over R. Alamouti's code corresponded to the quaternion algebra over R, which is the only division algebra over R. They went around this problem by considering the fields Q(x) in the field of real numbers. Here $x \in$ R is any transcendental number, for example π, e, $\zeta(3)$, ..

Cyclic division algebras

Cyclic division algebras can be constructed over Q(x) using

* a finite field extension F/k of degree n with cyclic Galois group Gal(F/k) = 'σ' and

* a certain element $\gamma \in \kappa^x$.

As a k-vector space, the algebra is expressible as $D = \bigoplus_{i=0}^{n-1} Fu^i$ where $u^n = \gamma$.

There is a well-known embedding of such a D into $M_n(F)$ that sends $f_0 + f_1 u + \cdots f_{n-1} u^{n-1}$ to

$$\begin{bmatrix} f_0 & \gamma\sigma(f_{n-1}) & \gamma\sigma^2(f_{n-2}) & \cdots & \gamma\sigma^{n-1}(f_1) \\ f_1 & \sigma(f_0) & \gamma\sigma^2(f_{n-1}) & \cdots & \gamma\sigma^{n-1}(f_2) \\ f_2 & \sigma(f_1) & \sigma^2(f_0) & \cdots & \gamma\sigma^{n-1}(f_3) \\ f_3 & \sigma(f_2) & \sigma^2(f_1) & \cdots & \gamma\sigma^{n-1}(f_4) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{n-2} & \sigma(f_{n-3}) & \sigma^2(f_{n-4}) & \cdots & \gamma\sigma^{n-1}(f_{n-1}) \\ f_{n-1} & \sigma(f_{n-2}) & \sigma^2(f_{n-3}) & \cdots & \sigma^{n-1}(f_0) \end{bmatrix}$$

By taking k to be various subfields of C containing Q(S) (the field generated by the elements of S over Q) in this formulation, and for each such F taking various k and $\tilde{a}$, a wide variety of space-time codes can be constructed for a wide range of signal sets. For further simplicity of construction, particularly in the selection of the element γ above, the authors of chose all their base fields k to contain transcendental elements; in most cases, their cyclic extensions F/k

were of the form $F_0(x) / k_0(x)$, where F_0/k_0 is a cyclic extension of number fields, and x is a transcendental. In these cases, the authors' construction yielded codes $X : Sn2 \rightarrow M_n(C)$.

Role of Mathematicians

What is the role of mathematicians in this field?

The subject is clearly very mathematical; yet, unlike classical coding theory which now has a mathematical life of its own and can, for instance, be thought of as a theory of subspaces of vector spaces over finite fields, the "center of gravity" of space-time codes currently lies very solidly in engineering. There is as yet no deep independent "mathematics of space-time codes": the driving force behind the subject consists of fundamental engineering problems that need to be solved before MIMO wireless communication reaches its full practical potential.

We, therefore, believe that, as things stand now, isolated mathematical investigations of space-time codes that are not grounded in concrete engineering questions would very likely lead to sterile results. At least for now, mathematicians can best contribute to the subject by working in collaboration with engineers who are motivated by fundamental engineering questions. This author, Bharath has found that the leading engineers in the field have a practical and intuitive understanding of much abstract mathematics, but welcome help from trained mathematicians.

There is clearly a lot of work for mathematicians to do.

References

Bharath A. Sethuraman,

"Division algebras and wireless communication,"

Notices of the AMS, vol. 57, pp. 1432–1439, Dec. 2010

RESEARCH PAPER PRESENTATION

Exploratory Data Analysis of Top 5 Crypto Currencies

Ms. Sakshi Rane**, Ms. Shreya Nadkarni*&Ms.Sumaiya Shaikh*

VPM's B.N. Bandodkar College of Science (Autonomous), Thane

Shreyanadkarni95105@gmail.com| sakshirane170403@gmail.com|sashaikh@vpmthane.org

Abstract : Crypto currency is the new age digital money which has impacted the financial world Prediction of future prices of crypto currencies using past data is possible with the help of statistical tools and techniques. The aim of this research is divided into two parts. The first part is to determine correlation or association between different crypto currency prices (Bitcoin, Ethereum, Binance coin, Tether, and Ripple), measured using statistical techniques Pearson correlation coefficients and to forecast the prices of the different crypto currencies (Bitcoin, Ethereum, Binance coin, Tether, and Ripple) using Facebook prophet since the significance of forecasting can be deemed one of the essential factors when gaining an edge in any kind of investment. The other part of the research deals with the risk and returns associated with these top 5 currencies many investors looking forward to a correct opportunity to invest in this new age digital money which is now a commodity.

Keywords : *Correlation, Time series analysis, Price Prediction, commerce.*

Introduction

Today, there are different type of crypto currencies with different technologies and functionality. Since their inception in 2008, 'crypto currencies' have become one of the most interesting and perhaps most misunderstood phenomena of the early 21st century. Since the inception of crypto currencies, an increasing number of financial institutions are getting involved in crypto currency trading. It is therefore important to summaries existing research papers and results on crypto currency trading.

Related work has discussed or partially examined literature related to the exchange of crypto currency. Greaves, A. and Au, B.,(2015) gave a brief survey of crypto currency systems using the bit coin transaction graph to predict the price of bit coin. Pichl, L. and Kaizoji, T., 2017 explored cost-effective trading opportunities by doing volatility analysis of bit coin prices using time series. The findings of this related work focused on specific field in crypto currency, including crypto currencies and crypto currency market introduction, crypto currency systems / platforms, bit coin literature review, etc.

This paper provides a comprehensive survey of crypto currency trading research, by covering Correlation analysis which measures the strength of relationship between the currencies ad Time series analysis which is the technique of studying and analysing the time series data collected over a period of time in order to study the trend in the data and extract important characteristics from it.

Objectives

The research project objectives are proposed as follows:

Objective 1: The first objective is to source the price data for Bit coin, Ethereum, Binance Coin, Tether, and Ripple crypto currencies.

Objective 2: Then second most important objective entails the exploration of different price distributions for each of the crypto currencies to identify the trends and to what magnitude (positive/negative) are the different crypto currency prices correlated with each other, measured using statistical techniques (Pearson correlation coefficients) and to predict the prices of the currencies to help smart investing for investors?

Objective 3: This is the final stage which would provide insights about risk and returns associated with these currencies.

Research Methodology
Data Collection

This study is based on secondary data which has been collected from the website https://www.kaggle.com/ nshreya788/eda-on-top-5-cryptocurrencies .The dataset contains 5 sheets namely as BTC, ETH, BNB, USDT, XRP. The data attributes are as follows:

1. Date : Date of observation

2. Close : Price on the given day in USD (Also the closing price for that day)

3. Open : Opening price on the given day

4. High : Highest price on the given day
5. Low : Lowest price on the given day
6. Volume : Volume of transactions on the given day

The data from the last five years (from the 9th November, 2017 to 31st December 2021) were included in the study.

Preliminary Work

Data Pre-processing

Data pre-processing for all crypto currencies was done in Google Collab notebook (Python). Once the data was sourced and collected, the dataset was then imported to Google Collab notebook (python) using the most effective data structure and data analysis (pandas) package toolkit, the existing data was first analysed before any cleaning, put within a data-frame. The closing price distribution of each of the crypto currencies was plotted on a graph with a year wise gap using (Plotly) library installed, to analyse the trend amongst these crypto currencies. Later, a correlation matrix heat map for all of these crypto currencies was generated. This was done by configuring the environment with the data visualization library (seaborn) based on matplotlib. For stationarity, statistics and predictive analytics a python library called Facebook Prophet was used.

Expected Outcomes

Analysis and Inference

Exploratory Data analysis entails the exploration of different price distributions to analyse any trends and seasonality. A Price distribution chart gives a glimpse of how the prices have been thought for the past 5 years for all crypto currencies.

Comparison of Closing Price Distributions of Bitcoin, Ethereum, Binance Coin, Tether and Ripple
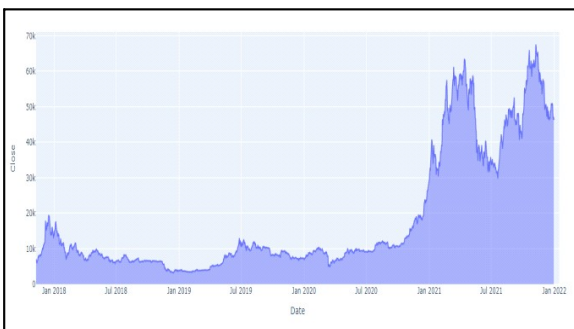
Fig.1. Closing price distribution of Bitcoin (BTC)
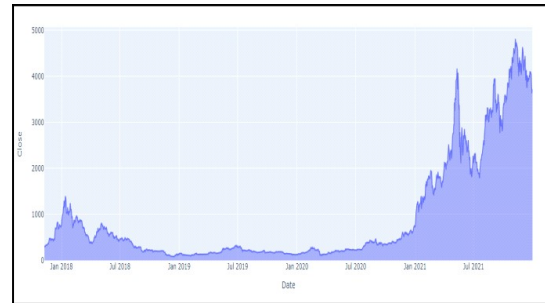
Fig.2. Closing price distribution of Ethereum (ETH)
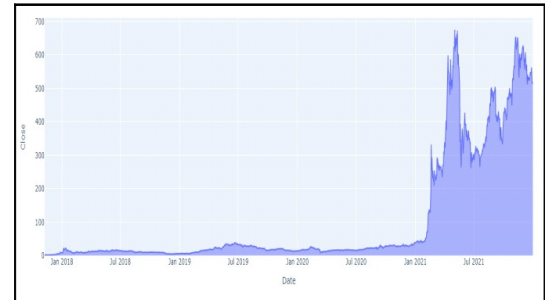
Fig.3. Closing price distribution of Binance Coin (BNB)
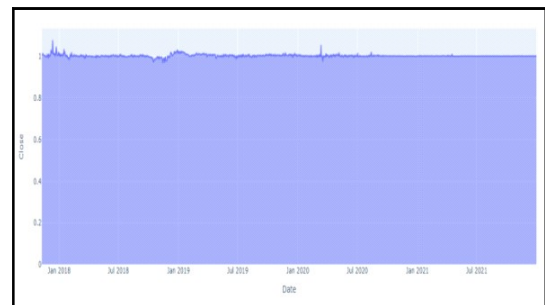
Fig.4. Closing price distribution of Tether (USDT)
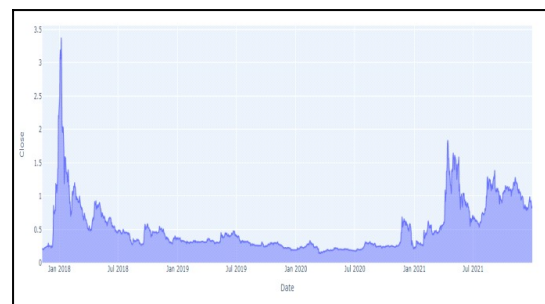
Fig.5. Closing price distribution of Ripple (XRP)

Insights

1. As an example, Bitcoin shows peak closer to December 2017, this is when the price of Bitcoin was $20,000 and skyrocketed to almost $65,000 over the last 5 years. Bitcoin(BTC) is leading the crypto market.

2. Similarly, The price of Ethereum (ETH) booms in late 2021.

3. Ripple (XRP) shows fluctuating price distributions.

Comparison Of The Average Closing Price Distribution Of Bitcoin, Ethereum, Binance Coin, Tether And Ripple

Year	BTC average price	ETH average price	BNB average price	Usdt average price	XRP average price
2017	12283.936	532.086680	3.483208867	1.01001769	0.5752066
2018	7572.29894	483.5077266	11.17253701	1.000030	0.6662736
2019	7395.24628	181.7701374	20.09297886	1.0048532	0.30970113
2020	11116.3780	307.54297	21.350414	1.001315	0.258398959
2021	47436.93202	2778.35414	378.30497793	1.000530	0.8680094

18
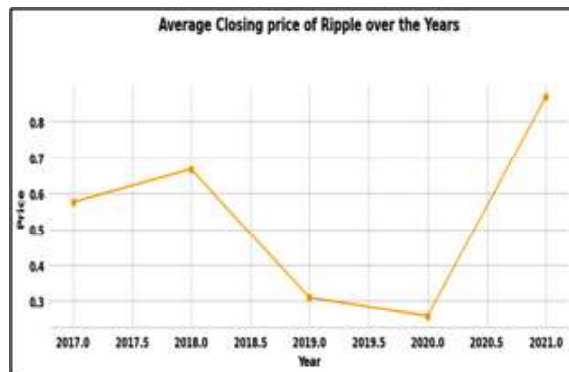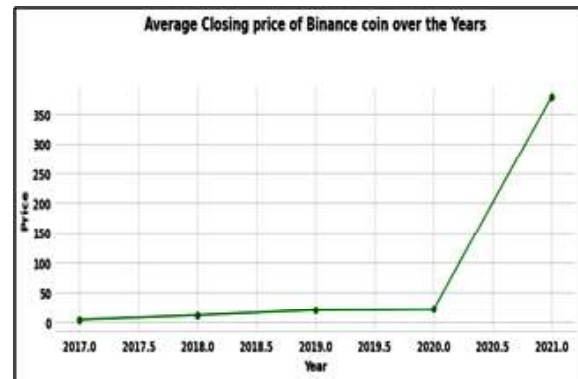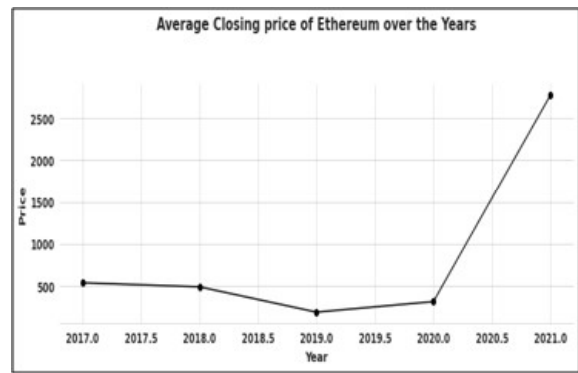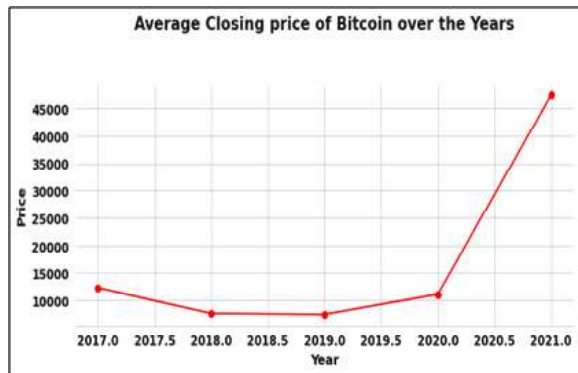
Insights

- Here we have taken into consideration the Average Closing prices (in USD) of Bit coin, Ethereum, Binance Coin, Tether and Ripple from the year 2017 to the year 2021.

- An average Price distribution chart gives a glimpse of how the average of prices have been thought for the last five years for all crypto currencies. As an example, Bit coin shows peak closer to $10000 in December 2017, this is when the price of Bit coin skyrocketed to almost $45,000.

- Similarly, other crypto currencies reached their peak closer to this period. However, Binance Coin

(BNB)'s chart shows average periodic rise over the years, whereas in case of Tether all price directions are completely on a downside.

Comparison of Transaction Volume of Bitcoin, Ethereum,

Binance Coin, Tether And Ripple

Crypto trading volume indicates interest in a crypto currency. The more people are buying and selling something, the higher the volume, which can drive even more interest in that particular crypto currency. This bar graph depicts how does Crypto trading volume has changed over the last 5 years for all five currencies.
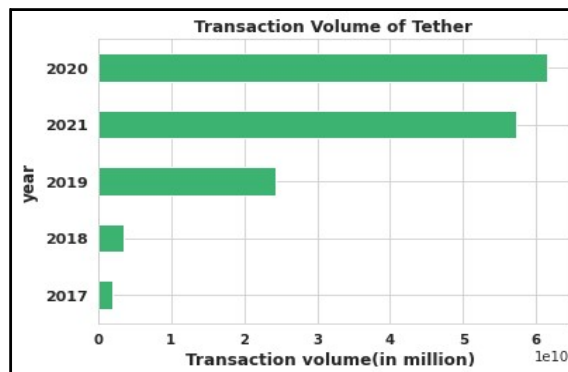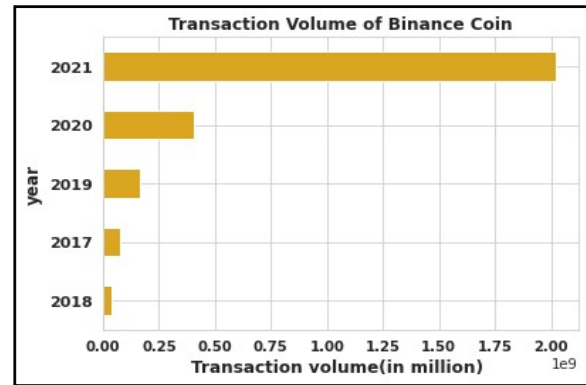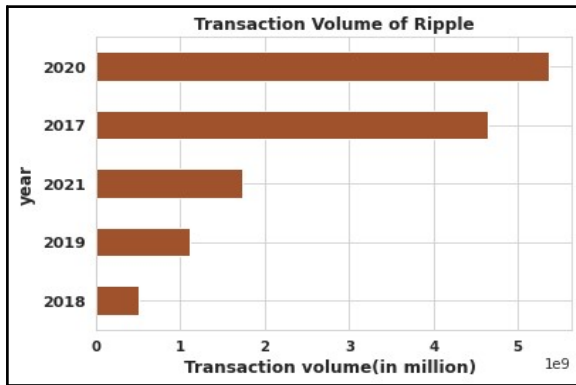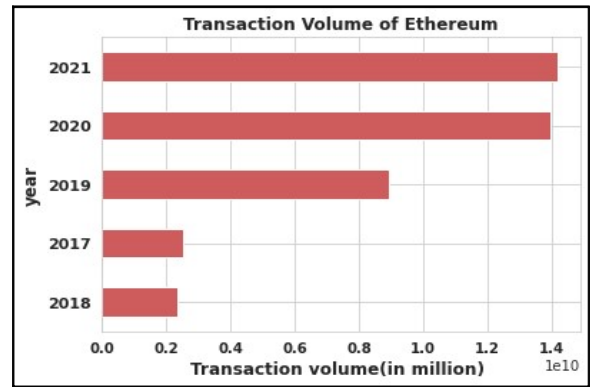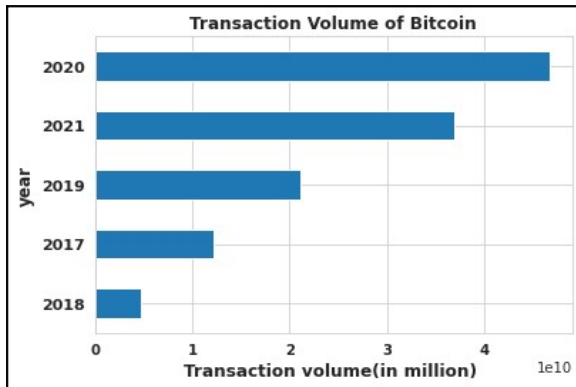
Insights

1) Highest transaction volume of Bit coin (BTC) had been observed for the year 2020.

2) Since it has been observed from the Closing Price Distribution graphs that Ethereum is a high beta version of binance coin. Ethereum (ETH) and Binance Coin (BNB) both are have having highest transaction volume in year 2021.

3) Highest trading volume for Tether had been recorded for year 2020.

4) Highest transaction volume for Ripple had been recorded in year 2020 followed by 2017

5) It has been observed from the above graphs that greater the volume of crypto currency transactions leads to fair crypto currency prices.

Correlation Analysis (Pearson Correlation)

Correlation attempt to discover the relationship, patterns, extensive connections, and developments among variables or datasets. There is a positive correlation among variables when an increase in a single variable, results in an increase in the other. On the other hand, a negative correlation means that after one variable increases, the other decreases and vice-versa.

Pearson Correlation is one of the most popular of the correlation statistic techniques

The formula used to calculate correlation coefficient is:

$\rho xy = Cov(x, y) / (\sigma x * \sigma y)$
where: ρxy = Pearson productmoment correlation coefficient
Cov (x, y) = covariance of variables x and y

σx = standard deviation of x

σy = standard deviation of y

Price values for Bit coin, Ethereum, Binance Coin, Tether and Ripple :

Correlation matrix of Five selected crypto currencies

The table below provides some guidelines for how to describe the strength of correlation coefficients

Correlation Coefficient (ρ)	Description
ρ = +1	Perfect Positive Correlation
ρ = -1	Perfect Positive Correlation
ρ = 0	No correlation
-1 < ρ < -0.8	Strong Negative Correlation
-0.8 < ρ < -0.5	Moderate Negative Correlation
-0.5 < ρ < 0	Weak negative correlation
0 < ρ < 0.5	Weak Positive correlation
0.5 < ρ < 0.8	Moderate Positive Correlation
0.8 < ρ < 1	Strong Positive Correlation

Insights

The above given heatmap shows that:

• Ethereum (ETH) is both highly correlated with binance coin (BNB) and more volatile than Binance coin (BNB). To borrow the lingo of equity markets, this makes Ethereum a high beta version of Binance coin. When Binance coin prices rise, Ethereum prices tend to rise more. When Binance coin prices fall, Ethereum prices tend to fall even further.

• Correlation analysis for Bit coin (BTC), Ethereum (ETH), Binance Coin (BNB), Tether (USDT) and Ripple (XRP) shows that all the currencies except Tether (USDT) are positively correlated to each other.

Benefits to the Society

• Crypto currency is the new age digital money which has impacted the financial world, since the launch of its first crypto-coin called Bit coin. The reason crypto currencies are getting popular is due to their various advantages ranging from being fast, secure, scalable, trustworthy, reliable, decentralized and more.

• With crypto currency, the dealing price is low to nothing at all—unlike, for example, the fee for transferring money from a digital wallet to a bank

account. one can easily make transactions at any time of the day or night, and there aren't any limits on purchases and withdrawals. And anyone is liberal to use crypto currency, instead of opening a bank account, which needs documentation and different paperwork.

- Funds transfer among parties can be smooth without the need of the third parties like credit/ debit cards or banks. It is a less expensive as compared to different online transactions Payments are secure and secured and provide an unparalleled stage of anonymity.

- The rise and fall of prices in the stock market is commonplace. Similarly, it's more volatile / speculative with the crypto currencies. Prices vary between 40-80 percent giving a huge factor of precariousness amongst its investors due to its volatile nature. Therefore, the correlation analysis and price prediction of these crypto currencies are often checked by regular investors for smart investing and hence, the returns are to be considered huge which crypto currency investors. The advantage of this project is to help these investors with better investment opportunities, using the correlation analysis and prediction of the top 5 crypto currencies.

- Volume can show the direction and movement of the crypto currency as well as a prediction of future price and its demand. Volume is an extremely important indicator for traders to determine the future profitability of crypto currencies.

Future Scope

Price Prediction of Bitcoin, Ethereum, Binance Coin, Tether and Ripple for 150 Days:

This prediction is done using the Facebook Prophet.

Facebook Prophet

FB Prophet is an open-source library published by Facebook. Prophet is a procedure for forecasting time series data based on an additive model where non-linear trends are fit with yearly, weekly, and daily seasonality, plus holiday effects. It works best with time series that have strong seasonal effects and several seasons of historical data.

➢ **Bitcoin (BTC) → Ethereum (ETH)**
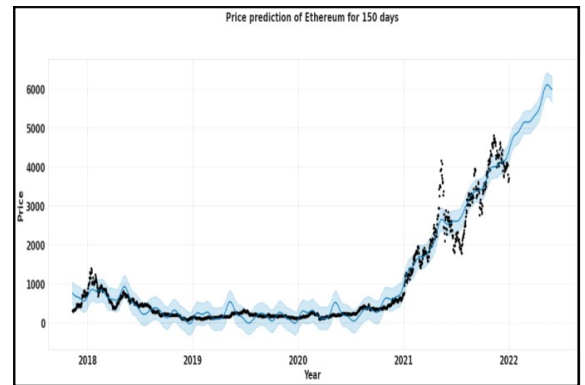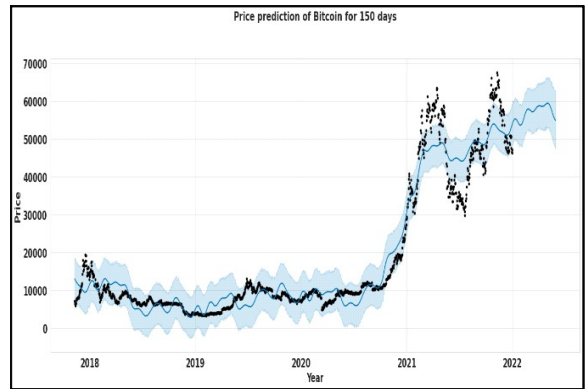
➢ **Binance Coin (BNB) → Tether (USDT)**
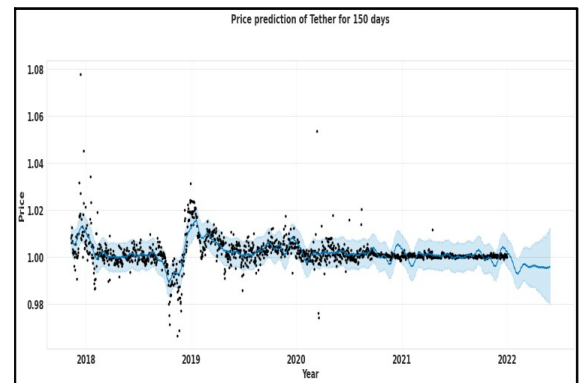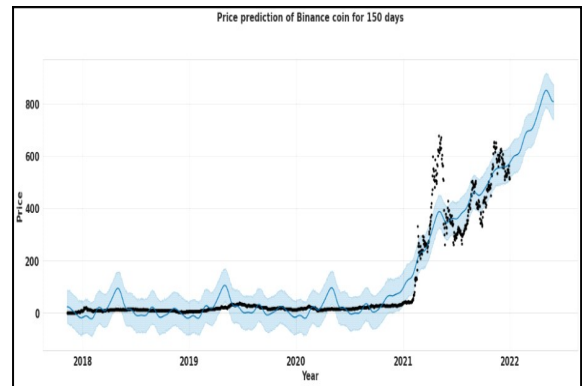
RIPPLE (XRP)
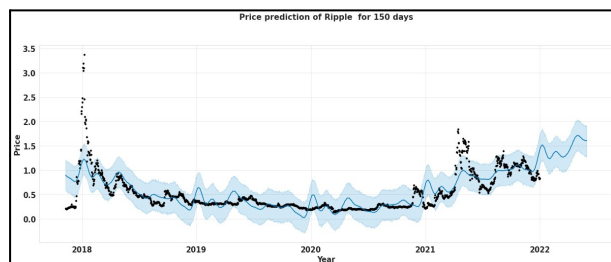
Insights

1. The reason for using Facebook prophet over other algorithms is because crypto currencies are very speculative and Fb prophet provides us with upper and lower limits of the predicted price which might be very helpful to investors.

2. In the graphs shown above black line depicts the actual price values, while this blue line depicts the predicted price values for next 150 days while this light blue Shady area is the upper limit and the lower limit in which prices of currencies might fall.

3. The predictive analysis indicates that the prices (in USD) will show a increasing trend for Bit coin (BTC), Ethereum (ETH), Binance Coin (BNB) and Ripple (XRP) for the next 150 days of year 2022. Where as the prices (in USD) for Tether (USDT) will continue to remain stable for the next 150 days of year 2022.

Risk and Return Analysis of 5 Currencies
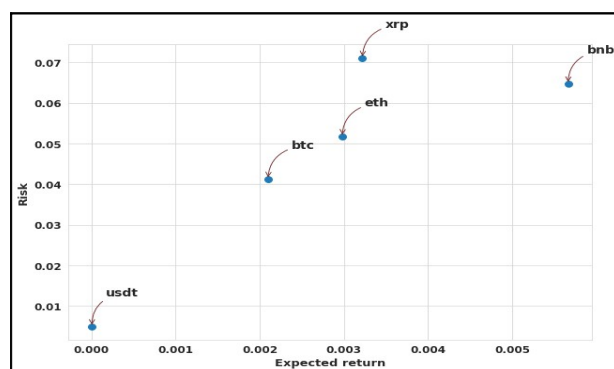
Insights
High return comes with High risk!

Average and standard deviation of the price of currencies have been calculated to assess the risk associated with returns. Standard deviation helps determine market volatility or the spread of asset prices from their average price. When prices move wildly, standard deviation is high, meaning an investment will be risky. Low standard deviation means prices are calm, so investments come with low risk.

From this graph concludes that although Ripple (XRP) had very high returns compared to any other currencies, but it is that much risky to invest money in Ripple (XRP) and Binance coin (BNB) as well. Closing Price distribution of Tether (USDT) had stable graph throughout the years hence, it has comparatively very less risk and return.

Swoc Analysis / Limitations

The following figure illustrate the SWOC (Strength, Weakness, Opportunities, Challenges) analysis of Five Currencies :
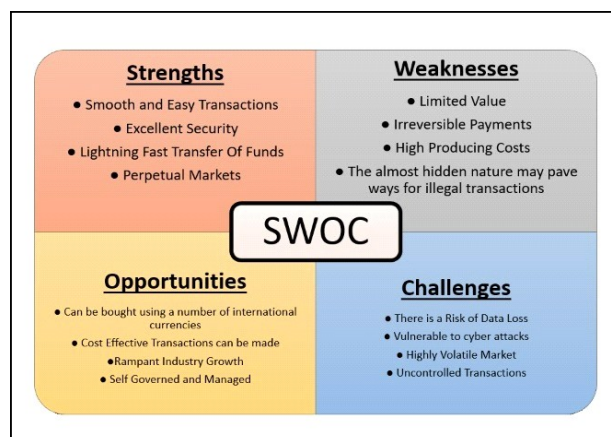
Conclusion

- The project objectives have been successfully achieved. Dataset have been exploratory analysed in order to identify the trends for each of the crypto currencies. Line plots helps us know the movement of the price and with the application of the necessary techniques to analyse it further for the prediction. The price distribution charts are a good method of exploring the data to understand how the distributions exist within the dataset.

- However, correlation coefficients are very handy in terms of understanding the relationships between different currencies. All crypto currencies were more or less correlated with each other positively or negatively. The highest positive correlation strength within all statistical models were Ethereum (ETH) and Binance Coin (BNB). Correlation

analysis for Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNB), Tether (USDT) and Ripple (XRP) shows that all the currencies except Tether (USDT) are positively correlated to each other. While Tether (USDT) is negatively correlated the currencies. This helps us understand that the price movements across crypto currencies are somewhat correlated with each other and hence they follow a similar trend when moving upwards or downwards. Correlation analysis gives valuable information on relationships of variables hence. This data can be used for example, to take advantage of changing market.

• Volume is such an important metric when analysing cryptos and it can help investors in showing a coin's direction. A greater volume of crypto currency transactions leads to fair crypto currency prices and removes the chance of distorted pricing. Investors can also use trading volume to recognize the fruitful opportunities of crypto market which can help them to reap enormous benefits.

• Bit coin is leading the market. Bit coin is probably the best-known crypto currency, and its value has been skyrocketed in recent years. Ethereum is another popular crypto currency worth looking into, as its value has been steadily increasing over the past. Ripple is another exciting option that could potentially see a lot of growth in 2022.

• Bit coin is a decentralized digital currency which can be classified more as an investment asset than a currency. Bit coin's growth and importance in the economy has not been gone unnoticed by the economic literature, where a growing interest around bitcoin arises. Previous literature shows that bit coin is a very volatile asset but an asset that is also valuable for risk management.

References

• Article on crypto currency from forbes https://www.forbes.com/advisor/investing/top-10-cryptocurrencies/

• Binance Coin official webpage https://www.binance.com/en

• Bitc oin official webpage: https://www.bitcoin.com/

• Crypto currency trading a comprehensive survey on research gate https://www.researchgate.net/publication/34

• Ethereum official webpage https://www.ethereum.org/, https://www.ethereum.org/ether

• Greaves, A. and Au, B., 2015. Using the bit coin transaction graph to predict the price of Bit coin

• Pichl, L. and Kaizoji, T., 2017. Volatility Analysis of Bit coin Price Time Series.

• Primary Data Source: https://finance.yahoo.com/cryptocurrencies/

• Official Webpage of Ripple https://ripple.com/

• Official webpage of Tether https://tether.to/en/

• Volatility Analysis of Bitcoin Price Time Series (aimspress.com)

Fuzzy Inventory Model with Advertisement and Selling Price Dependent Demand under Inflation with Variable Production for Deteriorating Items

Tanzim S. Shaikh

Department of Statistics, University of Mumbai
E-mail: tanzimsshaikh@gmail.com

Dr. S. P. Gite

Associate Professor
Department of Statistics, University of Mumbai
E-mail: santgite@yahoo.com

Abstract : Prime objective of this paper is to construct a fuzzy inventory model that takes into consideration the impact of inflation and the value of money in time to acquire the optimum order inventory cycle and the minimum overall average costs. The developed production inventory model where the demand is advertisement and selling price dependent and it has a direct linear impact on the production rate. With the initial production rate and subsequent variation in production rate, inventory level increases in the first two stages. After the accumulation of inventory, deterioration occurs, which follows two parameters of Weibull distribution. The shortages are not considered. In the fuzzy model, hexagonal fuzzy numbers are used to derive optimum solution and defuzzification by the graded mean integration representation method. A numerical example is given to demonstrate the proposed model's applicability, and carried out sensitivity analysis to reveal the impact of a change in parameter values.

Keywords: *Weibull distribution deterioration, Variable Production rate, hexagonal fuzzy number, Advertisement and Selling Price Dependent Demand, inflation.*

1. Introduction

Inventory describes a substantial stock of items that are kept readily available to ensure the organized, effective, and efficient operation of the market. The demand for a product is the major factor influencing a market's ups and downs. A product's demand may be influenced by several number of factors, including the time of year, the region, advertisements, trends, promotions etc. In this study, an advertisement and selling price dependent demand has been considered. Demands of this nature are typical of newly released products on the market. Inventory analysis frequently considers how quickly things deteriorate. Inventory models for deteriorating products have been studied by many inventory modelers. But not all types of goods, like meat, bread, milk, and so forth, start to disintegrate as soon as they are produced; instead, they preserve their freshness for a while before doing so.

In this production inventory model, the influence of inflation is considered for deteriorating products. In the current unstable economy, the inflationary effects cannot be underestimated, especially for long-term investments because the ordering strategy could also be affected by uncertainty over rising inflation. The consequence of inflation should be taken into account because it discredits currency. In inventory models, inflation was first investigated by (Buzacott, 1975).

Under the effect of inflation and time value of money (Bose et al., 1995) developed an inventory model where shortages were considered and demand was linearly dependent on time for deteriorating items. (Ghasemi, 2015) studied economic production quantity models with condition that holding cost is a function of the ordering run length.(Geetha & Udayakumar, 2016) studied an ordering policy with demand dependent on advertisement and price and shortages are partially allowed for deteriorating items. (Ardak & Borade, 2017) suggested production inventory model to obtain economic production quantity with variable deterioration rate and mixed demand rate. deteriorating items. Imperfect production inventory model was investigated by (Manna et al., 2017) where demand depends on advertisement and the producing defective units' rate is a function of production rate.(Sinha & Modak, 2019) formed a new approach for developing the production inventory model by considering the fact of controlling the carbon emission by production houses.(Hasan & Mashud, 2019) developed an inventory model based on demand which is selling price, frequency of

advertisement and time dependent for deteriorating items with partially backlogged shortages.(Rathore, 2019) studied inventory model with reliability consideration for non-instantaneous deteriorating items where demand is advertisement dependent. (Shekhawat et al., 2021) developed comprehended review paper about the advances and recent developments in the field of inventory supply chain for deteriorating items for the year 2016-2019.

(K. Kumar et al., 2021) studied an inventory policy for medicinal products which deteriorates with three parameter Weibull distribution and partly backlogged shortages.(Singh & Rani, 2021) developed an economic production quantity model with markdown policy with constant deteriorating rate under inflationary condition where shortages are allowed.(A. Barman et al., 2022) investigated an optimal inventory policy for non-instantaneous deteriorating products with a manufacturer – retailer supply chain model considering both cases of with and without shortages. (Malumfashi et al., 2022) developed an economic production inventory model where production rate has two phases for deteriorating items with exponential demand without shortages.

In any inventory system, there are multiple types of uncertainties. i.e., deterioration, changing set-up cost, holding cost, ordering cost etc. For the above uncertainties, it becomes more convenient to deal such problems with fuzzy set theory. Fuzzy set theory was first introduced by (Zadeh, 1965).(Roy et al., 2009) studied inventory model with fuzzy inflation for a deterioratingitem where demand is stock dependent. Keeping inflation under consideration (Pal et al., 2014) developed production inventory model in fuzzy environment for deteriorating items with ramp type demand. Also, (Pal et al., 2015) developed the fuzzy economic production quantity model considering the inflation and ramp type demand where shortages are allowed. For deteriorating items (Biswas & Islam, 2019) formulated production inventory model under fuzzy approach with demand function of population, selling price and frequency of advertisement. (Sen & Saha, 2020) investigated inventory model under fuzzy approach considering negative exponential demand for deteriorating items. (H. Barman et al., 2021) developed production inventory model under cloud-fuzzy environment for deteriorating items where demand depends on time with shortages. A production inventory model was developed by (S. Kumar, 2021) where demand rate dependents on time exponentially and partial shortages are allowed for deteriorating items.

In this proposed paper, under a fuzzy approach, production inventory model for deteriorating items is built, to discover the idyllic inventory cycle time and the minimum possible total average costs. Demand has a direct, linear impact on the production rate. With starting production rate, followed by rise in production rate, inventory level rises in the first two stages. A lower production rate in the initial stage helps to prevent having many items in stock, which reduces holding costs and thus provides a way to ensure customer happiness and maximizing profit. In the third stage, after inventory accumulates, deterioration occurs which is two-parameter Weibull deterioration. Shortages are not considered. The implementation of the presented model is demonstrated using a numerical example and sensitivity analysis.

2. Definition and Preliminaries
Definition 1.

A fuzzy set Ã on the given universal set is a set of order pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x) : x \in X\}$, where, $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ is a mapping called membership function. The membership function is also a degree of compatibility or a degree of truth of x in Ã.

Definition 2. (D. Dutta, 2012)

The α-cut of Ã is defined by,

$$A_{\alpha} = \{x : \mu_{\tilde{A}}(x) = \alpha, \alpha \geq 0\}$$

If R is a real line, then a fuzzy number is a fuzzy set Ã with membership function $\mu_{\tilde{A}} : X \rightarrow [0, 1]$, having following properties,

i. Ã is normal i.e., there exists $x \in$ R such that $\mu_{\tilde{A}}(x) = 1$

ii. Ã is piecewise continuous

iii. $\sup p(\tilde{A}) = cl\{ x \in$ R $: \mu_{\tilde{A}}(x) > 0\}$

iv. Ã is a convex fuzzy set.

Definition 3. (Nayak et al., 2021)

The fuzzy number set $\tilde{A} = (a, b, c, d, e, f)$ where, $a \leq b \leq c \leq d \leq e \leq f$ and defined on R, is called the Hexagonal fuzzy number, if the membership function of Ã is given by,

$$\mu_{\tilde{A}}(x) = \begin{cases} L_1(x) = \frac{1}{2}\left(\frac{x-a}{b-a}\right), & a \le x \le b \\ L_2(x) = \frac{1}{2} + \frac{1}{2}\left(\frac{x-b}{c-d}\right), & b \le x \le c \\ 1, & c \le x \le d \\ R_1(x) = 1 - \frac{1}{2}\left(\frac{x-d}{e-d}\right), & d \le x \le e \\ R(x) = \frac{1}{2}\left(\frac{f-x}{f-e}\right), & e \le x \le f \\ 0, & Otherwise \end{cases}$$

The α-cut of Ã = (a, b, c, d, e, f), $0 \le \alpha \le 1$ is

A(α) = $[A_L(\alpha), A_R(\alpha)]$ where,

$A_{L_1}(\alpha) = a + (b-a)\alpha = L_1^{-1}(\alpha)$

$A_{L_2}(\alpha) = b + (c-b)\alpha = L_2^{-1}(\alpha),$

$A_{R_1}(\alpha) = e + (e-d)\alpha = R_1^{-1}(\alpha),$

$A_{R_2}(\alpha) = f + (f-e)\alpha = R_2^{-1}(\alpha),$

And,

$$L^{-1}(\alpha) = \frac{L_1^{-1}(\alpha) + L_2^{-1}(\alpha)}{2} = \frac{a+b+(c-a)\alpha}{2}$$
$$R^{-1}(\alpha) = \frac{R_1^{-1}(\alpha) + R_2^{-1}(\alpha)}{2} = \frac{e+f+(d-f)\alpha}{2}$$

Definition 4 (Nayak et al., 2021)

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two hexagonal fuzzy numbers, then arithmetical operations are defined as,

1. $\tilde{A} \oplus \tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6)$

2. $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4, a_5 b_5, a_6 b_6)$

3. $\tilde{A} - \tilde{B} = (a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4, a_5-b_5, a_6-b_6)$

4. $\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}, \frac{a_5}{b_5}, \frac{a_6}{b_6}\right)$

5. $\alpha \oplus \tilde{A} = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha a_5, \alpha a_6) & \alpha \ge 0 \\ (\alpha a_6, \alpha a_5, \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1) & \alpha < 0 \end{cases}$

Definition 5.

If Ã = (a, b, c, d, e, f) is a hexagonal fuzzy number, then the graded mean integration representation

(GMIR) method of Ã is defined as,

$$P(\tilde{A}) = \frac{\int_0^{W_A} \frac{h}{2}\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right) dh}{\int_0^{W_A} h\, dh}, \text{ with } 0 \le W_A \le 1.$$

$$P(\tilde{A}) = \frac{a + 3b + 2c + 2d + 3e + f}{12}$$

3. Notations and assumptions

The following notations and assumptions are considered throughout the paper:

3.1 Notations

α   :   Demand coefficient

β   :   Demand constant

S   :   Selling-price

T   :   Duration of cycle

r   :   Discount rate which represents time value of money

i   :   Inflation rate per unit time

C_{HC} :   Holding cost

C_{PC} :   Production cost

C_{DC} :   Deterioration cost

SC   :   Setup cost

Q_1 :   Maximum inventory level at time t_1

Q_2 :   Maximum inventory level at time t_2

$I_1(t)$:   Inventory level, at any time t, during $[0, t_1]$

$I_2(t)$:   Inventory level, at any time t, during $[t_1, t_2]$

$I_3(t)$:   Inventory level, at any time t, during $[t_2, T]$

TC(T)   :   Total inventory cost

$\tilde{C}_{HC}$:   Fuzzy holding cost

$\tilde{C}_{DC}$:   Fuzzy deterioration cost

$\tilde{C}_{PC}$:   Fuzzy Production cost

$\widetilde{SC}$:   Fuzzy Setup cost

Assumptions

1. Single product is considered for the inventory cycle.

2. The demand rate $D(s)$ is dependent on advertisement cost A per unit time and selling price$s$ i.e .$D(s) = A^m \alpha s^{-\beta}$ where,

   $\alpha > 0, \beta > 0, 0 \leq m \leq 1$, where, α is scaling factor, β is index of price elasticity, m is shape parameter.

3. The production rate is linearly dependent on demand, that is,

   $P(s) = \eta \, D(s)$ where, $\eta > 1$ and production rate is greater than demand rate D(s).

4. Lead time is zero.

5. Shortages are not considered and backlogged are not allowed.

6. Manufactured products are added to the stock.

7. The deterioration of the products starts after a certain fix time. The rate of deterioration at time (t_2, T) is $(t) = \gamma \lambda t^{\lambda-1}$ which is two parameter Weibull distribution where, α represents scale parameter and β represents shape parameter. There is no deterioration before time t_2.

4. Model formulation

As shown in Figure 1, the inventory cycle starts with zero stock level at t = 0. Two rates of production are considered under advertisement and selling price-dependent demand. In the first and second production time interval $(0, t_1)$ and (t_1, t_2) respectively, there is no deterioration. During the time interval $(0, t_1)$ production rate is $\eta \, D(s)$, thus, inventory starts to build at the rate of $(n-1)D(s)$ units. level of inventory reaches Q_1 at the time t = t₁, then in the time interval (t_1, t_2), inventory levels start rising at a rate $\theta(\eta-1)D(s)$. A lower production rate in the initial stage helps to prevent having a large number of inventory products, which reduces holding costs and provides a method for maintaining customer satisfaction and maximizing profit. Production is terminated at time t = t₂ when the inventory level reached Q_2. Due to deterioration and demand, the inventory level begins to decline and ultimately falls to zero at time t = T. since, there are no shortages allowed, the cycle ends.
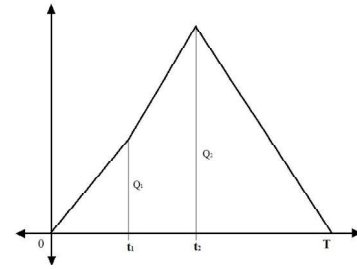
Fig.1. Graphical representation of inventory model.

The differential equations representing the inventory model are,

$$\frac{dI_1(t)}{dt} = (\eta - 1)D(s) \qquad 0 \leq t \leq t_1 \qquad (1)$$

$$\frac{dI_2(t)}{dt} = \theta(\eta - 1)D(s) \qquad t_1 \leq t \leq t_2 \qquad (2)$$

$$\frac{dI_3(t)}{dt} + \theta(t)I_3(t) = -D(s) \qquad t_2 \leq t \leq T \qquad (3)$$

Where, $(t) = \gamma \lambda t^{\lambda-1}$

The boundary conditions are,

$$I_1(0) = 0, I_1(t_1) = I_2(t_1) = Q_1, I_2(t_2) = I_3(t_2) = Q_2, I_3(T) = 0 \quad (4)$$

The solutions of differential equations above are given by,

$$I_1(t) = \frac{(\eta - 1)A^m \alpha t}{s^\beta} \qquad (5)$$

$$I_2(t) = \frac{\theta(\eta - 1)A^m \alpha t}{s^\beta} + \frac{(1 - \theta)(\eta - 1)A^m \alpha t_1}{s^\beta} \qquad (6)$$

$$I_3(t) = \frac{\eta}{p^\gamma}(t_d - t) + \frac{\eta}{p^\gamma}\left[(T - t_d) + \frac{\alpha}{\beta + 1}(T^{\beta+1} - t_d^{\beta+1})\right]e^{-\alpha t_d^\beta} \quad (7)$$

$$I_4(t) = \frac{\eta}{p^\gamma}\left[(T - t) + \frac{\alpha}{\beta + 1}(T^{\beta+1} - t^{\beta+1})\right]e^{-\alpha t^\beta} \qquad (8)$$

Using initial boundary conditions,

$$Q_1 = \frac{(\eta - 1)A^m \alpha t_1}{s^\beta} \qquad (9)$$

$$Q_2 = \frac{(\eta - 1)A^m \alpha t_1}{s^\beta} + \frac{(\eta - 1)(t_2 - t_1)A^m \alpha \theta}{s^\beta} \qquad (10)$$

The different costs included in total cost are as follows, considering the influence of inflation and time value of money,

Total cost of inventory cycle per unit time is,

$TC = \frac{1}{T}$ [Production Cost + Holding Cost + Deterioration Cost + Setup Cost]

i) Production Cost $= C_{PC} \left[\int\limits_0^{t_1} \frac{A^m \alpha \eta}{s^\beta} e^{-(r-i)t} dt + \int\limits_{t_1}^{t_2} \frac{A^m \alpha \eta \theta}{s^\beta} e^{-(r-i)t} dt \right]$

$$= C_{PC} \cdot \frac{A^m \alpha \eta}{s^\beta} \left[\theta \left(t_2 - \frac{(r-i)t_2^2}{2} \right) + (1-\theta) \left(t_1 - \frac{(r-i)t_1^2}{2} \right) \right] \quad (11)$$

Holding Cost $= C_{HC} \cdot \left[\int\limits_0^{t_1} I_{1(t)} e^{-(r-i)t} dt + \int\limits_{t_1}^{t_2} I_2(t) e^{-(r-i)t} dt + \int\limits_{t_2}^{T} I_3(t) e^{-(r-i)t} dt \right]$

$$= C_{HC} \cdot \left\{ \begin{array}{l} \frac{A^m \alpha (\eta - 1)}{s^\beta} \left(\frac{\theta t_2^2}{2} - \frac{(r-i)t_1^3}{3} - \frac{\theta(r-i)t_2^3}{3} + \frac{\theta(r-i)t_1^3}{3} \right) \\[2mm] + \frac{A^m \alpha (\eta - 1)(1-\theta)}{s^\beta} \left(t_1 t_2 - \frac{t_1^2}{2} + \frac{(r-i)t_1^3}{2} - \frac{(r-i)t_1 t_2^2}{2} \right) \\[2mm] + \frac{A^m \alpha}{s^\beta} \left[\frac{(T - t_2)^2}{2} + \frac{\gamma \lambda (T^{\lambda+2} - t_2^{\lambda+2})}{(\lambda+1)(\lambda+2)} - \frac{\gamma T t_2 (T^\lambda - t_2^\lambda)}{(\lambda+1)} - (r-i) \left(\frac{T^3}{6} - \frac{T t_2^2}{2} + \frac{t_2^3}{3} \right) \right. \\[2mm] \left. \qquad\qquad - \frac{\gamma(r-i)T^{\lambda+1}(T^2 - t_2^2)}{2(\lambda+1)} + \frac{\gamma(r-i)(T^{\lambda+3} - t_2^{\lambda+3})}{(\lambda+1)(\lambda+3)} \right] \end{array} \right\} \quad (12)$$

iii) Deterioration Cost $= C_{DC} \left[\int\limits_{t_2}^{T} \theta(t) \cdot I_3(t) e^{-(r-i)t} dt \right]$

$$= C_{DC} \left\{ \frac{A^m \alpha \gamma \lambda}{s^\beta} \left[\frac{T^{\lambda+1}}{\lambda(\lambda+1)} - \frac{T t_2^\lambda}{\lambda} + \frac{t_2^{\lambda+1}}{(\lambda+1)} + \frac{(r-i)(T^{\lambda+2} - t_2^{\lambda+2})}{\lambda+2} - \frac{(r-i)T(T^{\lambda+1} - t_2^{\lambda+1})}{\lambda+1} \right] \right\} \quad (13)$$

Setup Cost = SC $\qquad\qquad (14)$

Then, the Total cost per unit time for inventory cycle is,

$$TC(T) = \frac{1}{T} \left[\begin{array}{l} C_{PC} \cdot \frac{A^m \alpha \eta}{s^\beta} \left[\theta \left(t_2 - \frac{(r-i)t_2^2}{2} \right) + (1-\theta) \left(t_1 - \frac{(r-i)t_1^2}{2} \right) \right] \\[2mm] + C_{HC} \cdot \left\{ \begin{array}{l} \frac{A^m \alpha (\eta-1)}{s^\beta} \left(\frac{\theta t_2^2}{2} - \frac{(r-i)t_1^3}{3} - \frac{\theta(r-i)t_2^3}{3} + \frac{\theta(r-i)t_1^3}{3} \right) \\[2mm] + \frac{A^m \alpha (\eta-1)(1-\theta)}{s^\beta} \left(t_1 t_2 - \frac{t_1^2}{2} + \frac{(r-i)t_1^3}{2} - \frac{(r-i)t_1 t_2^2}{2} \right) \\[2mm] + \frac{A^m \alpha}{s^\beta} \left[\frac{(T-t_2)^2}{2} + \frac{\gamma \lambda (T^{\lambda+2} - t_2^{\lambda+2})}{(\lambda+1)(\lambda+2)} - \frac{\gamma T t_2 (T^\lambda - t_2^\lambda)}{(\lambda+1)} - (r-i) \left(\frac{T^3}{6} - \frac{T t_2^2}{2} + \frac{t_2^3}{3} \right) \right. \\[2mm] \left. \qquad - \frac{\gamma(r-i)T^{\lambda+1}(T^2 - t_2^2)}{2(\lambda+1)} + \frac{\gamma(r-i)(T^{\lambda+3} - t_2^{\lambda+3})}{(\lambda+1)(\lambda+3)} \right] \end{array} \right\} \\[2mm] + C_{DC} \left\{ \frac{A^m \alpha \gamma \lambda}{s^\beta} \left[\frac{T^{\lambda+1}}{\lambda(\lambda+1)} - \frac{T t_2^\lambda}{\lambda} + \frac{t_2^{\lambda+1}}{(\lambda+1)} + \frac{(r-i)(T^{\lambda+2} - t_2^{\lambda+2})}{\lambda+2} - \frac{(r-i)T(T^{\lambda+1} - t_2^{\lambda+1})}{\lambda+1} \right] \right\} + SC \end{array} \right] \quad (15)$$

Let $t_1 = c_1 T$, $t_2 = c_2 T$ such that, $0 < c_1, c_2 < 1$ and $T > t_2 > t_1$

28

$$TC(T) = \frac{1}{T}\left[\begin{array}{l} C_{PC}.\frac{A^m \alpha \eta}{s^\beta}\left[\theta\left(t_2 - \frac{(r-i)(c_2^2 T^2)}{2}\right) + (1-\theta)\left(t_1 - \frac{(r-i)(c_1^2 T^2)}{2}\right)\right] \\[2mm] + C_{HC}.\left\{\begin{array}{l} \frac{A^m \alpha(\eta-1)}{s^\beta}\left(\frac{\theta(c_2^2 T^2)}{2} - \frac{(r-i)(c_1^3 T^3)}{3} - \frac{\theta(r-i)(c_2^3 T^3)}{3} + \frac{\theta(r-i)(c_1^3 T^3)}{3}\right) \\[2mm] + \frac{A^m \alpha(\eta-1)(1-\theta)}{s^\beta}\left(c_1 c_2 T^2 - \frac{(c_1^2 T^2)}{2} + \frac{(r-i)(c_1^3 T^3)}{2} - \frac{(r-i)c_1 T(c_2^2 T^2)}{2}\right) \\[2mm] + \frac{A^m \alpha}{s^\beta}\left[\begin{array}{l} \frac{(T-c_2 T)^2}{2} + \frac{\gamma\lambda\left(T^{\lambda+2}-(c_2^{\lambda+1}T^{\lambda+1})\right)}{(\lambda+1)(\lambda+2)} - \frac{\gamma c_2 T^2\left(T^\lambda-(c_2^\lambda T^\lambda)\right)}{(\lambda+1)} \\[2mm] -(r-i)\left(\frac{T^3}{6} - \frac{T(c_2^2 T^2)}{2} + \frac{(c_2^3 T^3)}{3}\right) \\[2mm] -\frac{\gamma(r-i)T^{\lambda+1}\left(T^2-(c_2^2 T^2)\right)}{2(\lambda+1)} + \frac{\gamma(r-i)\left(T^{\lambda+3}-(c_2^{\lambda+3}T^{\lambda+3})\right)}{(\lambda+1)(\lambda+3)} \end{array}\right] \end{array}\right\} \\[2mm] + C_{DC}\left\{\frac{A^m \alpha\gamma\lambda}{s^\beta}\left[\begin{array}{l} \frac{T^{\lambda+1}}{\lambda(\lambda+1)} - \frac{T(c_2^\lambda T^\lambda)}{\lambda} + \frac{(c_2^{\lambda+1}T^{\lambda+1})}{(\lambda+1)} \\[2mm] + \frac{(r-i)\left(T^{\lambda+2}-(c_2^{\lambda+2}T^{\lambda+2})\right)}{\lambda+2} - \frac{(r-i)T(T^{\lambda+1}-(c_2^{\lambda+1}T^{\lambda+1}))}{\lambda+1} \end{array}\right]\right\} + SC \end{array}\right] \tag{16}$$

By minimizing the total cost TC(T), the following differential equation can be solved to determine the optimum value of T, $\frac{dTC(T)}{dT} = 0$ satisfying the condition, $\frac{d^2TC(T)}{dT^2} > 0$

Fuzzy Model

Due to uncertainty in the market, all parameters cannot be defined precisely, hence considering

$\tilde{C}_{PC} = (C_{PC1}, C_{PC2}, C_{PC3}, C_{PC4}, C_{PC5}, C_{PC6})$, $\tilde{C}_{HC} = (C_{HC1}, C_{HC2}, C_{HC3}, C_{HC4}, C_{HC5}, C_{HC6})$,

$\widetilde{SC} = (SC_1, SC_2, SC_3, SC_4, SC_5, SC_6)$, $\tilde{C}_{DC} = (C_{DC1}, C_{DC2}, C_{DC3}, C_{DC4}, C_{DC5}, C_{DC6}$ are Hexagonal fuzzy numbers.

In a fuzzy sense, the total cost of the model per unit of time is given by,

$$\widetilde{TC}(T) = \frac{1}{T}\left[\begin{array}{l} \tilde{C}_{PC}.\frac{A^m \alpha \eta}{s^\beta}\left[\theta\left(t_2 - \frac{(r-i)(c_2^2 T^2)}{2}\right) + (1-\theta)\left(t_1 - \frac{(r-i)(c_1^2 T^2)}{2}\right)\right] \\[2mm] + \tilde{C}_{HC}\left\{\begin{array}{l} \frac{A^m \alpha(\eta-1)}{s^\beta}\left(\frac{\theta(c_2^2 T^2)}{2} - \frac{(r-i)(c_1^3 T^3)}{3} - \frac{\theta(r-i)(c_2^3 T^3)}{3} + \frac{\theta(r-i)(c_1^3 T^3)}{3}\right) \\[2mm] + \frac{A^m \alpha(\eta-1)(1-\theta)}{s^\beta}\left(c_1 c_2 T^2 - \frac{(c_1^2 T^2)}{2} + \frac{(r-i)(c_1^3 T^3)}{2} - \frac{(r-i)c_1 T(c_2^2 T^2)}{2}\right) \\[2mm] + \frac{A^m \alpha}{s^\beta}\left[\begin{array}{l} \frac{(T-c_2 T)^2}{2} + \frac{\gamma\lambda\left(T^{\lambda+2}-(c_2^{\lambda+1}T^{\lambda+1})\right)}{(\lambda+1)(\lambda+2)} - \frac{\gamma c_2 T^2\left(T^\lambda-(c_2^\lambda T^\lambda)\right)}{(\lambda+1)} \\[2mm] -(r-i)\left(\frac{T^3}{6} - \frac{T(c_2^2 T^2)}{2} + \frac{(c_2^3 T^3)}{3}\right) \\[2mm] -\frac{\gamma(r-i)T^{\lambda+1}\left(T^2-(c_2^2 T^2)\right)}{2(\lambda+1)} + \frac{\gamma(r-i)\left(T^{\lambda+3}-(c_2^{\lambda+3}T^{\lambda+3})\right)}{(\lambda+1)(\lambda+3)} \end{array}\right] \end{array}\right\} \\[2mm] + \tilde{C}_{DC}\left\{\frac{A^m \alpha\gamma\lambda}{s^\beta}\left[\begin{array}{l} \frac{T^{\lambda+1}}{\lambda(\lambda+1)} - \frac{T(c_2^\lambda T^\lambda)}{\lambda} + \frac{(c_2^{\lambda+1}T^{\lambda+1})}{(\lambda+1)} \\[2mm] + \frac{(r-i)\left(T^{\lambda+2}-(c_2^{\lambda+2}T^{\lambda+2})\right)}{\lambda+2} - \frac{(r-i)T(T^{\lambda+1}-(c_2^{\lambda+1}T^{\lambda+1}))}{\lambda+1} \end{array}\right]\right\} + \widetilde{SC} \end{array}\right] \tag{17}$$

Let $\widetilde{TC}_i(T)$ be the corresponding total cost obtained by replacing $\widetilde{SC}_i, \tilde{C}_{HCi}, \tilde{C}_{DCi}, \tilde{C}_{PCi}$ in equation (17) for i=1, 2, 3, 4, 5, 6. Using graded mean representation to defuzzify the fuzzy total cost $\widetilde{TC}(T)$

We get,

$$\widetilde{TC}(T) = \frac{1}{12}[\widetilde{TC}_1(T) + 2\widetilde{TC}_2(T) + 3\widetilde{TC}_3(T) + 3\widetilde{TC}_4(T) + 2\widetilde{TC}_5(T) + \widetilde{TC}_6(T)]$$

By minimizing the total cost $\widetilde{TC}(T)$, the following differential equation can be solved to determine the optimum value of T.

$$\frac{d\widetilde{TC}(T)}{dT} = 0 \text{ satisfying the condition, } \frac{d^2\widetilde{TC}(T)}{dT^2} > 0$$

5 Numerical Examples

5.1 Crisp Model

Consider following parametric values.

C_{PC} = ₹ 10/unit, C_{DC} = ₹ 12/unit,

C_{HC} = ₹ 3/unit, SC = ₹ 600/order,

$\alpha = 40000$, $\beta = 2.5$, $s = 20$, $m = 0.4$,

$A = 150$, $\gamma = 0.01$, $\lambda = 1$, $b = 1.1$, $\eta = 4$,

$c_1 = 0.2$, $c_2 = 0.3$, $r = 0.5$, $i = 1.2$

The solution of crisp model is,

T = 0.906073, TC(T) = 3207.06,

t_1 = 0.1812, t_2 = 0.2718.

5.2 Fuzzy Model

$\alpha = 40000$, $\beta = 2.5$, $s = 20$, $m = 0.4$,

$A = 150$, $\gamma = 0.01$, $\lambda = 1$, $b = 1.1$, $? = 4$,

$c_1 = 0.2$, $c2 = 0.3$, $r = 0.5$, $i = 1.2$,

$\tilde{C}_{PC} = (7, 8, 9, 11, 12, 13)$,

$\tilde{C}_{HC} = (1, 2, 3, 4, 5, 6)$,

$\tilde{C}_{DC} = (9, 10, 11, 13, 14, 15)$,

$\widetilde{SC} = (300, 400, 500, 600, 700, 800)$,

The solution of fuzzy model is given by,

$T = 0.843407$, $\widetilde{TC}(T) = 3194.15$,

$t_1 = 0.1687$, $t_2 = 0.2530$.

Graphic comparisons are available between the two models' outcomes. Figure 2 shows a comparison

between the Time parameters, Figure 3 demonstrates a comparison between the ideal total cost values.
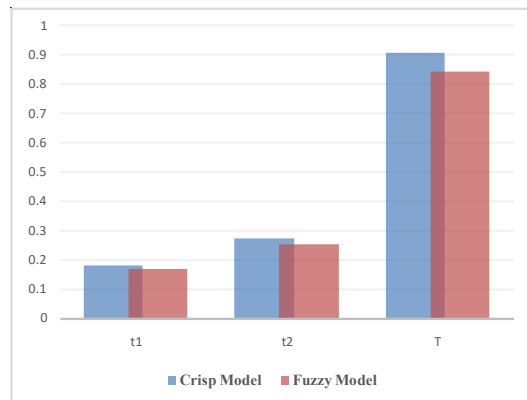
Fig. 2 Comparison of Time parameters
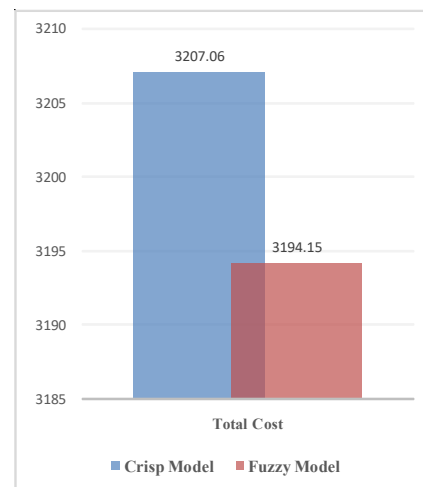
Fig. 3 Comparison of Total cost

Figure 2 illustrates that the crisp model has longer production times (t1 and t2) and optimal inventory cycle times (T) than the fuzzy model. Figure 3 shows that the overall cost for the fuzzy model is lower than the crisp model. As a result, the fuzzy model is advantageous since it lowers costs as order volume rises, increasing profit.

6 Sensitivity Analysis

Considering the above numerical example of the fuzzy model for sensitivity analysis to examine the impact of changing various inventory model parameters.

Table 1. Sensitivity analysis for various parameters

		T	TC	t_1	t_2
	110	0.8828	2895.81	0.1766	0.2648
	130	0.8614	3052.35	0.1723	0.2584
A	150	0.8434	3194.15	0.1687	0.2530
	170	0.8278	3324.35	0.1656	0.2483
	190	0.8142	3445.12	0.1628	0.2443
		T	TC	t_1	t_2
	8	0.8616	2745.03	0.1723	0.2585
	9	0.8424	2969.70	0.1705	0.2557
C_{PC}	10	0.8434	3194.15	0.1687	0.2530
	11	0.8346	3418.42	0.1669	0.2504
	12	0.8259	3642.49	0.1652	0.2478
		T	TC	t_1	t_2
	10	0.8439	3193.25	0.1688	0.2532
	11	0.8437	3193.70	0.1687	0.2531
C_{DC}	12	0.8434	3194.15	0.1687	0.2530
	13	0.8431	3194.61	0.1686	0.2529
	14	0.8429	3195.06	0.1686	0.2529
		T	TC	t_1	t_2
	1	1.1164	2937.8	0.2233	0.3349
	2	0.9672	3054.87	0.1934	0.2902
C_{HC}	3	0.8774	3150.99	0.1755	0.2632
	4	0.8141	3234.86	0.1628	0.2442
	5	0.7657	3310.37	0.1531	0.2297
	400	0.7480	3005.86	0.1496	0.2244
	500	0.8140	3133.83	0.1628	0.2442
SC	600	0.8709	3252.48	0.1742	0.2613
	700	0.9212	3364.05	0.1842	0.2764
	800	0.9664	3469.98	0.1933	0.2899
	20000	1.0806	1881.37	0.2161	0.3242
	30000	0.9368	2549.95	0.1874	0.2810
α	40000	0.8434	3194.15	0.1687	0.2530
	50000	0.7757	3822.94	0.1551	0.2327
	60000	0.7233	4440.83	0.1447	0.2170
	2.3	0.6707	5221.32	0.1341	0.2012
	2.4	0.7535	4069.45	0.1507	0.2261
β	2.5	0.8434	3194.15	0.1687	0.2530
	2.6	0.9408	2526.73	0.1882	0.2822
	2.7	1.0460	2015.82	0.2092	0.3138

As shown in Table 1, the data can be interpreted as,

1. An increase in advertisment cost A, increases the total average cost TC (T) but decreses production time (t_1 and t_2) and optimum inventory cycle time (T).

2. With the increase in the production cost (C_{PC}), it is observed that, optimum inventory cycle time (T), production time (t_1 and t_2) decreses, but the total average cost TC (T) increases.

3. An increase in deteriorating cost (C_{DC}) increases the total average cost TC (T) but decreses production time (t_1 and t_2) and optimum inventory cycle time (T).

4. With the increase in the holding cost (C_{HC}), it is observed that, optimum inventory cycle time (T), production time (t_1 and t_2) decreses, but the total average cost TC (T) increases.

5. An increase in the set-up cost (SC), increases optimum inventory cycle time (T), production time (t_1 and t_2), also the total average cost TC(T) increases.

6. An increase in α increases the total average cost TC (T) but decreses production time (t_1 and t_2) and optimum inventory cycle time (T).

7. With increase in the value of β, the total average cost TC(T) decreses but optimum inventory cycle time (T), production time t_1 and t_2 increses..

7 Conclusion

In the developed fuzzy production inventor model, demand is a function of advertisement and selling price under the effect of inflation and time value of the money, and the production rate is proportional to the demand rate. Inventory level increases with two production rates at two stages of inventory cycle. As the production ceases, the inventory level diminishes due to demand deterioration rate, which is following two parameter Weibull distribution. Production is started again instantly as the inventory level reaches zero. Shortages are not considered in this model.

For the crisp model and fuzzy model, optimum total average cost, total inventory cycle time(T) and production time (t1 and t2) are obtained. Fuzzy model is built using the hexagonal fuzzy numbers for certain parameters of the models and graded mean integration method is used for defuzzification. Shortages are not allowed. Both models, crisp model and fuzzy model, are compared. It can be observed that, for the fuzzy model, the optimum solution is much better than the optimum solution of crisp model.

In future aspect, one can develop this chapter by adding shortages with fully backlogging or with partial backlogging.

Acknowledgement

References

1) Ardak, P. S., &Borade, A. B. (2017). An EPQ Model with Varying Rate of Deterioration and Mixed Demand Pattern. International Journal of Mechanical and Production Engineering Research and Development, 7(6), 11-20. https://doi.org/10.24247/ijmperddec20172

2) Barman, A., Das, R., & De, P. K. (2022). An analysis of optimal pricing strategy and inventory scheduling policy for a non-instantaneous deteriorating item in a two-layer supply chain. Applied Intelligence, 52(4), 4626-4650. https://doi.org/10.1007/s10489-021-02646-2

3) Barman, H., Pervin, M., Roy, S. K., & Weber, G.-W. (2021). Back-ordered inventory model with inflation in a cloudy-fuzzy environment. Journal of Industrial & Management Optimization, 17(4), 1913-1941. https://doi.org/10.3934/jimo.2020052

4) Biswas, A. K., & Islam, S. (2019). A Fuzzy EPQ Model for Non-Instantaneous Deteriorating Items where Production Depends on Demand which is Proportional to Population, Selling Price as well as Advertisement. Independent Journal of Management & Production, 10(5), 1679-1703. https://doi.org/10.14807/ijmp.v10i5.897

5) Bose, S., Goswami, A., & Chaudhuri, K. S. (1995). An EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting. Journal of the Operational Research Society, 46(6), 771-782.

6) Buzacott, J. A. (1975). Economic order quantities with inflation. Journal of the Operational Research Society, 26(3), 553-558.

7) D. Dutta, D. D. (2012). Fuzzy Inventory Model without Shortage Using Trapezoidal Fuzzy Number with Sensitivity Analysis. IOSR Journal of Mathematics, 4(3), 32-37. https://doi.org/10.9790/5728-0433237

8) Geetha, K. V., & Udayakumar, R. (2016). Optimal Lot Sizing Policy for Non-instantaneous Deteriorating Items with Price and Advertisement Dependent Demand Under Partial Backlogging. International Journal of Applied and Computational Mathematics, 2(2), 171-193. https://doi.org/10.1007/s40819-015-0053-7

9) Ghasemi, N. (2015). Developing EPQ models for non-instantaneous deteriorating items. Journal of Industrial Engineering International, 11(3), 427-437. https://doi.org/10.1007/s40092-015-0110-1

10) Hasan, R., & Mashud, A. H. (2019). An Economic Order Quantity model for Decaying Products with the Frequency of Advertisement, Selling Price and Continuous Time Dependent Demand under Partially Backlogged Shortage. International Journal of Supply and Operations Management, 6(4), 296-314.

11) Kumar, K., Kumar, N., & Meenu, M. (2021). An inventory system for varying decaying medicinal products in healthcare trade. Yugoslav Journal of Operations Research, 31(2), 273-283. https://doi.org/10.2298/YJOR200125011K

12) Kumar, S. (2021). A Fuzzy type Backlogging Production Inventory Model for Perishable Items with Time Dependent Exponential Demand Rate. Journal of Ultra Scientist of Physical Sciences Section A, 33(4), 51-65. https://doi.org/10.22147/jusps-A/330403

13) Malumfashi, M. L., Ismail, M. T., & Ali, M. K. M. (2022). An EPQ Model for Delayed Deteriorating Items with Two-Phase Production Period, Exponential Demand Rate and Linear Holding Cost. Bulletin of the Malaysian Mathematical Sciences Society. https://doi.org/10.1007/s40840-022-01316-x

14) Manna, A. K., Dey, J. K., & Mondal, S. K. (2017). Imperfect production inventory model with production rate dependent defective rate and advertisement dependent demand. Computers & Industrial Engineering, 104, 9-22. https://doi.org/10.1016/j.cie.2016.11.027

15) Nayak, D. K., Routray, S. S., Paikray, S. K., & Dutta, H. (2021). A fuzzy inventory model for Weibull deteriorating items under completely backlogged shortages. Discrete & Continuous Dynamical Systems - S, 14(7), 2435. https://doi.org/10.3934/dcdss.2020401

16) Pal, S., Mahapatra, G. S., & Samanta, G. P. (2014). An EPQ model of ramp type demand with Weibull deterioration under inflation and finite horizon in crisp and fuzzy environment. International Journal of Production Economics, 156, 159-166. https://doi.org/10.1016/j.ijpe.2014.05.007

17) Pal, S., Mahapatra, G. S., & Samanta, G. P. (2015). A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness. Economic Modelling, 46, 334-345. https://doi.org/10.1016/j.econmod.2014.12.031

18) Rathore, H. (2019). An Inventory Model with Advertisement Dependent Demand and Reliability Consideration. International Journal of Applied and Computational Mathematics, 5(2), 33. https://doi.org/10.1007/s40819-019-0618-y

19) Roy, A., Maiti, M. K., Kar, S., & Maiti, M. (2009). An inventory model for a deteriorating item with displayed stock dependent demand under fuzzy inflation and time discounting over a random planning horizon. Applied Mathematical Modelling, 33(2), 744-759. https://doi.org/10.1016 j.apm.2007.12.015

20) Sen, N., & Saha, S. (2020). Inventory model for deteriorating items with negative exponential demand, probabilistic deterioration and fuzzy lead time under partial back logging. Operations Research and Decisions, 30(3). https://doi.org/10.37190/ord200306

21) Shekhawat, S., Rathore, H., & Sharma, K. (2021). Analysis of Deteriorating Inventory Work (2016-2019). Journal of Physics: Conference Series, 1913(1), 012101. https://doi.org/10.1088/1742-6596/1913/1/012101

22) Singh, S. R., & Rani, M. (2021). An EPQ model with life-time items with multivariate demand with markdown policy under shortages and inflation. Journal of Physics: Conference Series, 1854(1), 012045. https://doi.org/10.1088/1742-6596/1854/1/012045

23) Sinha, S., & Modak, N. M. (2019). An EPQ model in the perspective of carbon emission reduction. International Journal of Mathematics in Operational Research, 14(3), 338-358. https://doi.org/10.1504/IJMOR.2019.099382

24) Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

An Efficient Generalized Index Calculus Algorithm

Arpit Vijay Raikwar[1*], Chandrashekhar Meshram[2], Khushal Devghare[3]

[1,2,3]Department of Post-Graduation Studies and Research in Mathematics
Jayawanti Haksar Government Post Graduation College,
College of Barkatullah University, Betul, M.P., India 460001
Corresponding author: Arpit Vijay Raikwar, raikwararpit@gmail.com

Abstract : In this paper, we present a generalised Index Calculus Algorithm for computing twin discrete logarithm problems in this study. We also offer a new variation of the discrete logarithm problem defined over a finite field in a multiplicative cyclic group. This is known as a twin discrete logarithm problem since it has two unique exponents. It has been determined that the twin discrete logarithm problem is more complex than the discrete logarithm problem. Some applications of the twin discrete logarithm problem (TDLP) in cryptography and network security are also investigated.

1. Introduction

The discrete logarithm problem DLP of the form $\alpha \equiv \beta^y \pmod{m}$ is very intractable problem in number theory, group theory and cryptography. As more calculations are done nowadays, and most scholars have started focusing on algorithmic challenges, the status of DLP began to rise in the twentieth century. DLP was invented in the 1950s and played an important role in cryptography and number theory at the time. Later, concepts of public key cryptography techniques emerged and were demonstrated on rotor machines utilising a cryptosystem based on shift-register sequences and DLP. The usage of DLP is important in shift-register sequences to locate the position of a block.

The US Digital Signature Algorithm is largely regarded as reliable because it is based on DLP rather than an integer factorization problem. It should be emphasised that DLP is more difficult to use for encryption than the integer factorization problem. Furthermore, it aids in the enforcement of export control regulations even when using a powerful encryption technique without degrading less stringently controlled digital signature systems. Although DLP and integer factorization problem are similar in that many techniques created for single issue can be adjusted to apply to the other issue, it has been discovered that DLP performs more effectively in some circumstances than Integer factorization problem. It is better to have more diversity, especially in terms of security. It should

be mentioned that numerous attempts have been made to design public key cryptography schemes using other concepts, such as pairing, lattices and polynomials, among others, which serve as the fundamental building blocks for the creation of cryptographic schemes.

Even in the topic of DLP complexity analysis, there have been various general assessments [1, 2, 3, 14], as well as more previous work on particular subfields. As a result, in this study, we will simply provide links to the most recent outcomes and share some high-level observations about the DLP's current position and potential future. Cheon [13] offered some auxiliary inputs for the discrete logarithm issue, which he then generalised in [17]. Meshram et al. [4, 5] demonstrated the application of DLP in the design of identity-based cryptographic schemes and GDLP in the design of identity-based cryptographic schemes and authentication methods in [6, 7, 8, 9, 10, 11].

Our contribution: As previously stated, the DLP plays a significant role in strengthen the security of cryptographic schemes. To improve the security of cryptographic schemes, we offer new DLP concepts such as Twin Discrete Logarithm Problem (TDLP), which works even when G is not a cyclic group, and we explore the difficulty of TDLP. We also suggested some changes to the Index Calculus Algorithm for computing TDLP. Furthermore, this TDLP is quite useful in addressing several issues in number theory, group theory, and so on, and it may be useful in wireless communication and adhoc networking.

Organization: The remaining sections are organised as follows: The TDLP is suggested in Section 2. Review of Index Calculus Algorithm based on the DLP was covered in Section 3. Section 4 establishes the principal result for TDLP complexity. Generalized Index Calculus Algorithm for computing TDLP is proposed in Section 5. Analysis the Generalized Index Calculus Algorithm proposed for computing TDLP is explained in Section 6. In section 7 we discussed some applications of TDLP for designing secure cryptographic scheme. Finally, we give a conclusion in Section 8.

2. Proposed TDLP

In this section, we cover the different types of DLP that have been proposed and discussed in the literature, including the TDLP. The domain of the other is the group of points on an elliptic curve over a finite field [16, 17] or the group of points on a hyper elliptic curve [18] over a finite field. Recently, the DLP has been divided into two types, one of which is a multiplicative group in a finite field, such as cyclic multiplicative group of the prime field. In the current study, we have expanded the idea of DLP to TDLP by employing cyclic multiplicative groups.

• **DLP :** Given a cyclic group G of order m, a primitive root β of the group element γ, the problem is to find an integer y such that $\alpha \equiv \beta^y (mod\ m)$ with $0 \leq y \leq m - 1$, this problem is called Discrete Logarithm Problem. In case if variable β is not the primitive root of cyclic group, the problem reduces to find a minimum integer, which satisfies $\alpha \equiv \beta^y (mod\ m)$.

It is fairly simple to calculate DLP using the exhaustive technique if is a small number. However, the exhaustive technique cannot be used to compute DLP if is a sufficiently large number. Now that the multiplicative group is defined over a finite field, we propose the TDLP. The formal definition is as follows:

• **TDLP:** Given a cyclic group G (it is not necessary that G be a cyclic group) of order $(m$ - $1)$, two elements β and γ of the group G, there exist element $\sigma \in G$, the problem is to find integers y and z satisfying the condition $\sigma \equiv \beta^y \gamma^z (mod\ m)$ with $1 \leq yz \leq m - 1$, s.t. $\beta \neq \gamma^i$ and $y \neq z_i$ then this is called TDLP. If variables β and γ are not the primitive roots of cyclic

group G, the problem reduces to find minimum integers y and z satisfying $\sigma \equiv \beta^y \gamma^z (mod\ m)$.

Compared to solving a problem with a single exponent of DLP, solving a problem with a double exponent of DLP is more challenging for obvious reasons. A more complicated formulation would be more useful in cryptography and network security.

3. Review of Index Calculus Algorithm

The Index Calculus Algorithm [15, 16] for computing DLP as a single exponent is covered in this section.

Given a generator β of a multiplicative cyclic group Z_n^* of order m and $\in Z_n^*$, the goal is to calculate the DLP, i.e. $y = log_\beta \alpha\ (mod\ m)$. The algorithm is as follows:

Pre-computation stage

1. Choose a factor base (a set of small primes that will be the primes considered during the algorithm): $A = k_1 k_2 k_3 \dots k_t$ of G such that a significant fraction of all elements in may be written effectively as a product of elements from A.

2. Collect linear relations involving the logarithms of A's constituents.

   2.1 Choose a random number $w, 0 \leq w \leq m - 1$ and calculate.

   2.2 Try to write β^w as a product of elements in

   $$A: \beta^w = \prod_{i=1}^{t} k_i^{e_i}, e_i \geq 0.$$

   If so, take the logarithm of the equation's both sides to get a linear relationship:

   $$w \equiv \sum_{i=1}^{t} e_i log_\beta k_i (mod\ m).$$

   2.3 Until $t + e$, where e is a small integer, such as $e = 10$, relations of the given form are found, repeat steps 2.1 and 2.2.

Calculation of DLP stage

3. To determine the values of $log_\beta k_i, 1 \leq i \leq t$ solve the linear system of $t + e$ equations (in t unknowns) gathered in step 2 working modulo.

Let choose a random integer $w, (1 \leq w \leq m - 2)$ and calculate a

$$\sigma \equiv \alpha\beta^w \pmod{m} \qquad (6)$$

By applying the condition of 4.2 of section 3, we get a congruence of the form

$$\alpha\beta^w \equiv k_1^{e_1}.k_2^{e_2}.k_3^{e_3} \ldots \ldots k_t^{e_t} \pmod{m} \qquad (7)$$

This implies

$$\alpha\beta^w \equiv \prod_{i=1}^{t} k_i^{e_i} \pmod{m}, e_i \geq 0 \qquad (8)$$

Equation (7) can be written equivalently as

$$log_\beta \alpha + w \, log_\beta \beta \equiv (e_1 \, log_\beta k_1 + e_2 \, log_\beta k_2 + \cdots + e_t \, log_\beta k_t) \, mod \, (m-1)$$

This implies

$$log_\beta \alpha + w \, log_\beta \beta \equiv \left(\sum_{i=1}^{t} e_i log_\beta k_i\right) mod \, (m-1)$$

$$\Rightarrow log_\beta \alpha + w \equiv \left(\sum_{i=1}^{t} e_i log_\beta k_i\right) mod \, (m-1) \qquad (9)$$

Since all terms in the equation (9), except $log_\beta \alpha$, it can be easily solved by using algorithm 3 for computing single exponent of DLP.

According the Index-Calculus Algorithm to solve TDLP, i.e. $\sigma \equiv \beta^y \gamma^z \pmod{m}$ under the conditions $\beta \neq \gamma^i, y \neq z_i$ in the multiplicative group defined over the nite eld of the order, we take the following three cases.

Case 1: We compute TDLP as follows after successfully completing the precipitation step (cf. section 3). Choose a random integer $w, (1 \leq w \leq m - 2)$ and compute

$$\beta \equiv \sigma (\beta\gamma)^w \pmod{m} \qquad (10)$$

where $\beta \neq \gamma^i$. Otherwise it will reduce to DLP.

Applying condition 1 of section 3 the factor base, we get a congruence of the form:

$$\sigma(\beta\gamma)^w \equiv k_1^{d_1}.k_2^{d_2}.k_3^{d_3} \ldots \ldots k_t^{d_t} \pmod{m} \qquad (11)$$

or

$$\sigma(\beta\gamma)^w \equiv \prod_{i=1}^{t} k_i^{d_i} \pmod{m}, d_i \geq 0 \qquad (12)$$

In equation (11) take log to the base on both the side, we obtain

$$log_\beta \sigma + w \, log_\beta \beta + w \, log_\beta \gamma$$
$$\equiv (d_1 log_\beta k_1 + d_2 log_\beta k_2 + d_3 \, log_\beta k_3 + \cdots \ldots \ldots + d_t \, log_\beta k_t) \, mod \, (m-1)$$

It reduces to

$$log_\beta \alpha + w\, log_\beta \beta + w\, log_\beta \gamma \equiv \left(\sum_{i=1}^{t} d_i\, log_\beta k_i\right) mod\ (m-1)$$

$$\Rightarrow log_\beta \alpha + w\, log_\beta \gamma \equiv \left(\sum_{i=1}^{t} d_i\, log_\beta k_i - w\right) mod\ (m-1) \qquad (13)$$

Since all terms in the aforementioned equation, with the exception of $log_\beta \gamma$ and $log_\beta \sigma$, are known, algorithm 3, which is used to compute a single exponent of DLP, may be used to quickly and easily solve it.

Case 2: In equation (11), take logarithm to the base δ on both the side, we get

$$log_\gamma \sigma + w\, log_\gamma \gamma + w\, log_\gamma \beta \equiv \left(d_1\, log_\gamma k_1 + d_2\, log_\gamma k_2 + \cdots + d_t\, log_\gamma k_t\right) mod\ (m-1)$$

After Simplifying the equation , we get

$$log_\gamma \alpha + w\, log_\gamma \gamma + w\, log_\gamma \beta \equiv \left(\sum_{i=1}^{t} d_i\, log_\gamma k_i\right) mod\ (m-1)$$

$$\Rightarrow log_\gamma \sigma + w\, log_\gamma \beta \equiv \left(\sum_{i=1}^{t} d_i\, log_\gamma k_i - w\right) mod\ (m-1) \qquad (14)$$

Again, all terms in the aforementioned equation, with the exception of $log_\beta \gamma$ and $log_\beta \sigma$, are known, hence it may be solved by using case 1.

Case 3: In equation (11) , take logarithm to the base α both the side, we have,

$$log_\sigma \sigma + w\, log_\sigma \beta + w\, log_\sigma \gamma \equiv \left(d_1\, log_\sigma k_1 + d_2\, log_\sigma k_2 + \cdots + d_t\, log_\sigma k_t\right) mod\ (m-1)$$

Which is equivalent to

$$log_\sigma \sigma + w\, log_\sigma \beta + w\, log_\sigma \gamma \equiv \left(\sum_{i=1}^{t} d_i\, log_\sigma k_i\right) mod\ (m-1)$$

$$\Rightarrow w\, log_\sigma \beta + w\, log_\sigma \gamma \equiv \left(\sum_{i=1}^{t} d_i\, log_\sigma k_i - 1\right) mod\ (m-1) \qquad (15)$$

Similarly, we should solve $log_\sigma \beta$ and $log_\sigma \gamma$.

As we discussed in the above three cases, we express that Index-Calculus Algorithm needs the double computation to compute TDLP which is more complex as compare to DLP in the finite cyclic group Z_n^* of the order $(m - 1)$.

6. Analysis of Generalized Index Calculus Algorithm

Based on the Index-Calculus Algorithm [15, 16] for computing DLP with a one exponent, we introduce its generalize for computing TDLP with two distinct exponents in the form of DLP in this section. We can see clearly from the TDLP that it works well because of the complexity generated by the two exponents, as opposed to DLP, which only deals with a single exponent. In cryptography, the difficulty of solving fundamental problems such as DLP, generalized DLP, integer factorization problems, pairing and lattices, and so on determines the security of a cryptographic scheme.

We now look at how TDLP can be used to create more secure cryptographic schemes. We now focus our attention one the following main formulation of TDLP i.e.

$$\sigma \equiv \beta^y \gamma^z (mod\ m) \quad (16)$$

We have twin discrete logarithm problem proposed under the condition $\beta \neq \gamma^i$ and $y \neq z_i$.

In case 1 stated in the proof of Theorem 2, it may show that in place of one unknown variable (involved in DLP), our goal is to find values of two knowns variable viz. $log_\beta \gamma$ and $log_\beta \sigma$. This will make the cryptographic system more secure in comparison of DLP.

Similarly, in case 2, we concluded that, instead of one unknown variable, we must compute two different unknowns. $log_\gamma \beta$ and $log_\gamma \sigma$ and it will work like to case 1. In case 3, Finally, we had two unknowns: $log_b \beta$ and $log_b \gamma$

It is very interesting to get that a choice of these six different unknowns can be used to develop any cryptographic schemes more secure and complex in comparison to DLP which deal with only one unknown by using Index-Calculus Algorithm.

2. Applications and Future Scope

In cryptography, group theory, and number theory, the discrete logarithm problem is a fundamental problem. It is vital for the security of cryptographic schemes such as public key cryptosystems, digital signature schemes, identity-based signature schemes, identity-based cryptosystems, elliptic curve cryptosystems, group-oriented encryption, homomorphism-based cryptosystems, and signature schemes, among others. The security of any cryptographic scheme is directly depend upon the difficulty of solving the fundamental problem used to design the cryptographic scheme. We show in this study that TDLP has two distinct exponents that are involved in the form of DLP. More secure cryptographic schemes, such as public key cryptosystems, digital signature schemes, identity-based signature schemes, identity-based cryptosystems, elliptic curve cryptosystems, group-oriented encryption, homomorphism-based cryptosystems, and signature schemes are easily designed.

3. Conclusion

In this paper, we proposed the Generalized Index-Calculus Algorithm for computing TDLP with two distinct exponents, as well as an extended version of DLP known as TDLP. In addition, we demonstrate that TDLP is more difficult and complicated than DLP. Theorem 5.1 establishes the modification to the Index-Calculus Algorithm for computing TDLP. We discussed the possibility of some cryptographic applications, such as public key cryptosystems, digital signature schemes, identity-based signature schemes, identity-based cryptosystems, elliptic curve cryptosystems, group-oriented encryption, homomorphism-based cryptosystems, and signature schemes, among other things based on TDLP which will improve the security of all cryptographic schemes. It is important to note that the application of this new aspect of TDLP has a compatibility with computers due to its two different components.

References

[1] A. OdlyzkoTop of FormBottom of Form, Discrete Logarithms: The Past and the Future, Towards a Quarter-Century of Public Key Cryptography, (2000), 59-75.

[2] A. Odlyzko, Discrete logarithms in nite elds and their cryptographic signicance, in Advances in Cryptology: Proceedings of Eurocrypt '84, Lecture Notes in Computer Science, Springer-Verlag, **209** (1985) 224–314.

[3] A. Odlyzko, Discrete logarithms and smooth polynomials, Finite Fields: Theory, Applications and Algorithms, Amer. Math. Soc., **168** (1994) 269–278.

[4] C Meshram and S. Meshram, An identity based cryptographic model for discrete logarithm and integer factoring based cryptosystem, Information Processing Letters. **113** (2013) 375-380.

[5] C Meshram, An Efficient ID-based Cryptographic Encryption based on Discrete Logarithm Problem and Integer Factorization Problem, Information Processing Letters. **115** (2015) 351-358.

[6] C. Meshram, S. A. Meshram and M. Zhang**,** An ID-based cryptographic mechanisms based on GDLP and IFP, Information Processing Letters, **112** (2012) 753–758.

[7] C Meshram and S. Meshram, An identity based Beta Cryptosystem, IEEE Proceedings of 7th International Conference on Information Assurance and Security (IAS 2011) Dec.5-8, (2011) 298-303.

[8] C. Meshram, Yuh-Min Tseng, Cheng-Chi Lee, S G Meshram, An IND-ID-CPA Secure ID-based Cryptographic Protocol using GDLP and IFP, INFORMATICA LITHUAN An International Journal, **28(3)** (2017), 471–484

[9] C. Meshram, Cheng-Chi Lee, Chun-Ta Li, Chin-Ling Chen, A Secure Key Authentication Scheme for Cryptosystems Based on GDLP and IFP, Soft Computing, **21 (24)** (2017), 7285-7291.

[10] C. Meshram, S. A. Meshram, Constructing New An ID-based Cryptosystem for IFP and GDLP based Cryptosystem. Journal of Discrete Mathematical Sciences and Cryptography, **20 (5)** (2017) 1121-1134.

[11] C Meshram, X Li, New Efficient Key Authentication Protocol for Public Key Cryptosystem using DL over Multiplicative Group, Journal of Information and Optimization Sciences, **39 (2)** (2018) 391–400.

[12] D. R. Stinson, Cryptography: Theory and Practice, CRC Press, Boc Raton, Florida, (1995).

[13] J. H. Cheon, Discrete logarithm problems and auxiliary inputs, Journal of Cryptology, **23** (2010) 457-476.

[14] K. S. McCurley, The discrete logarithm problem, Cryptography and Computational Number Theory, C. Proc. Symp. Appl. Math., Amer. Math. Soc., **42** (1990) 49–74.

[15] L. M. Adleman, A sub-exponential algorithm for discrete logarithm problem with applications to cryptography. Proceedings of the IEEE 20th Annual Symposium on Foundation of Computer Science, (1979) 55-60.

[16] M. E. Hellman, J. M. Reyneri, Fast computation of discrete logarithms in GF (q). Advanced in Cryptology-Proceedings of Crypto, **82**, (1983) 3-13.

[17] M. Kim, J. H. Cheon and I.S. Lee, Analysis on a generalized algorithm for the strong discrete logarithm problem with auxiliary inputs, Mathematics of Computation, **83** (2014) 1993-2004.

[18] N. Koblitz, Hyperelliptic cryptosystems, Journal of Cryptology, **1**(1989) 139-150.

[19] N. Koblitz, Elliptic curve cryptosystems, Mathematics of Computation, **48** (1987) 203-209.

[20] V. Miller, Use of elliptic curve in cryptography, Advances in Cryptology-CRYPTO vol.85, Lecture Notes in Computer Science, Springer-Verlag, **218**, (1986) 417-426.

Analysing Customer Patronage of Fast Food Restaurants in Urban Maharashtra

Ryanston Rodrigues

Student, Department of Statistics, University of Mumbai, Santacruz (E), Maharashtra
Email ID: ryanstonrodrigues2000@gmail.com

Abstract: Customer Patronage can be defined as the act of repeated buying by customers. It involves the purchase of goods and services by a person or business. In recent times, fast food consumption has become increasingly popular in a developing country like India due to several lifestyle changes and the introduction of several popular fast food brands. The aim of this research is to find out the important factors affecting the customer patronage of particular fast food restaurants in urban Maharashtra, namely McDonald's, KFC, Burger King, Subway, Domino's and Pizza Hut. Primary data has been collected for the research by purposive sampling method using online questionnaire. Descriptive statistics and inference techniques have been used to analyse the data. Factor analysis has been used to identify important factors affecting customer patronage to restaurants. Analysis of the data reveals that **Service & Atmosphere** and **Value for Money** are two key factors that should be taken into consideration while running the brick-and-mortar model of fast food restaurants in order to increase patronage by customers.

Keywords: Customer Patronage, Purposive Sampling, Factor Analysis, Brick-and-Mortar Model

Introduction

India is a country of varied cuisines and tastes. It is often said that taste and eating habits constantly change from one state to another. In recent times, fast food has become increasingly popular among Indian youth. Home cooked meals are now being replaced by so called 'junk food'. From burgers to pizza, such food is becoming one of the world's fastest growing food types. With the influence of western culture and cuisine and the impetus provided by urbanization and globalization, there has been a surge in the number of fast food restaurants popping up in major states and cities. Cities like Thane, Mumbai and Pune have become home to several fast food restaurants, from local ones to major multinational brands like Kentucky Fried Chicken (KFC) and McDonald's. These restaurants serve both vegetarian and non-vegetarian meals to cater to their customers. Another major reason people visit fast food restaurants is that there is a perceived sense of notion that such meals and their particular taste cannot be replicated at home. From seasonal menus to quirky themes and concepts to appease the Indian taste buds, these fast food restaurants are constantly improvising their strategies to attract more customers.

Objectives

- To identify the significant factors affecting customer patronage to fast food restaurants.
- To identify the consumption pattern of customers visiting fast food restaurants.
- To test the hypotheses regarding the amount spent by consumers.

Literature Survey

Fast food industry is an ever growing industry. According to a study by worldwatch.org, India's fast food industry is growing by 40 percent per year. With the liberalization policy that came into effect in the year 1991, several multinational brands set up restaurants in India. Kentucky Fried Chicken (KFC), McDonald's, Domino's and Pizza Hut all entered into the Indian market during the years 1995-1996. As per Anderson and Sullivan (1993), customer satisfaction leads to an increased tendency in customers to purchase from a particular brand. Chitnis (2019) states that the modern generation does not want to stick to the traditional stereotypes of cooking food at home, leading to an increased consumption of fast food. People also prefer a cozy atmosphere and a modern, urban vibe in such restaurants. Further, Kara et. al (1995) concluded in their study that variety, price and location are the important factors that young customers look out for in American restaurants. Food quality and menu variety also affect customer patronage. Since a major part of the Indian market includes vegetarian consumers, restaurants have to make sure to include appropriate food items that also do not hurt the religious sentiments of patrons.

Methodology

The study was conducted based on 6 popular fast food chain of restaurants viz. McDonald's, KFC, Burger King, Subway, Domino's and Pizza Hut. Since the study is a part of an exploratory data analysis (EDA), a sample size of 100 respondents was selected using purposive sampling method keeping in mind that the respondents had clear knowledge about the questions asked. A questionnaire was prepared which consisted of two parts: Demographics and Patronage Behaviour. A 5 point likert summative scale (Strong Disagree:1, Disagree:2, Indifferent:3, Agree:4, Strongly Agree:5) was used to collect Patronage data. Data was collected from residents of urban Maharashtrian cities like Mumbai, Thane and Pune who have access to fast food restaurants in their vicinity. Descriptive statistics and inference methods such as Fisher's Exact test were used to analyze the data. Factor analysis was performed to identify important factors. Analysis was carried out on R Studio Version 3.6.3.

Analysis and Results

The following variables were measured on the 5 point likert scale:

- Food Taste and Quality

- Menu Variety

- Quick and Prompt Service

- Ambience, Décor and Environment

- Restaurant Popularity

- Affordability

- Discounts/Offers

- Location and Accessibility

- Privacy

The mean score for each of the above variables is tabulated below:

Variable	N	Mean
Food Taste and Quality	100	3.810345
Menu Variety	100	3.801724
Quick and Prompt Service	100	3.594828
Ambience, Décor and Environment	100	3.62069
Restaurant Popularity	100	3.836207
Affordability	100	3.75
Discounts/Offers	100	3.362069
Location and Accessibility	100	3.931034
Privacy	100	3.103448

We observe that **Location and Accessibility**, **Restaurant Popularity** and **Food Taste and Quality** have the highest mean scores. This suggests that the above 3 factors affect customer patronage based on descriptive statistics.

The frequency and percentage tables and corresponding chart for the customer demographic data collected are as follows:

Age of Respondents

The mean age of respondents was found to be 21.54 years.

Gender of Respondents

	Frequency	Percent
Female Male Total	39	39.0
	61	61.0
	100	100.0

Educational Qualification of Respondents:

	Frequency	Percent
10th	10	10.0
12th	14	14.0
Diploma or Certificate	1	1.0
Graduate	57	57.0
Post Graduate	18	18.0
Total	100	100.0

Occupation of Respondents

	Frequency	Percent
Other	3	3.0
Private Sector	11	11.0
Public Sector	1	1.0
Self Employed	5	5.0
Student	80	80.0
Total	100	100.0

Economic Status of Respondents

	Frequency	Percent
Above	13	13.0
Average Affluent	1	1.0
Average Below	78	78.0
Average	5	5.0
Poor	3	3.0
Total	100	100.0

Preferred Fast Food Restaurant

	Frequency	Percent
Burger King	9	9.0
Domino's	21	21.0
KFC	10	10.0
McDonald's	33	33.0
Other	23	23.0
Pizza Hut	1	1.0
Subway	3	3.0
Total	100	100.0

McDonald's (33%) is found to be the most preferred brand of consumers, followed by **Domino's** (21%). McDonald's and Domino's are famous for their collection of burgers and pizza, which indicates that these two fast foods are popular among consumers. 23% of the respondents chose 'Other' as an option, which indicates that their preferred restaurant is different than the 6 restaurants mentioned.

Frequency of Visit

	Frequency	Percent
Often	24	24.0
Rarely	34	34.0
Regularly	5	5.0
Seldom	22	22.0
Very Rarely	15	15.0
Total	100	100.0

Preferred Mode of Payment

	Frequency	Percent
Card Payment	9	9.0
Cash Payment	41	41.0
Net Banking	3	3.0
UPI	47	47.0
Total	100	100.0

With the introduction of UPI technology for payment, which is both fast and convenient, it has become a popular mode of payment among consumers. However, cash payment also remains popular among consumers.

Preferred Time of Visit

	Frequency	Percent
Afternoon	21	1.0
Evening	60	60.0
Morning	1	1.0
Night	18	18.0
Total	100	100.0

Most consumers prefer to visit fast food restaurants in the evening. This may be attributed to the fact that people go to eat out after completing their daily work routine.

Reasons for Eating Out

	Frequency	Percent
Eating out with Family	10	10.0
Eating out with Friends	44	44.0
No Specific Reason	19	19.0
Special Occasions and Celebration	21	21.0
Work or Study Related	6	6.0
Total	100	100.0

Eating out with friends is found to be the most common reason to visit fast food restaurants while work or study related reasons are least observed.

Average Time Spent at a Fast Food Restaurant:

	Frequency	Percent
15 to 30 mins	28	28.0
30 to 45 mins	36	36.0
45 to 60 mins	20	20.0
Less than 15 mins	6	6.0
More than 60 mins	10	10.0
Total	100	100.0

Amount Spent Per Visit

	Frequency	Percent
1000-1500	7	7.0
1500-2000	4	4.0
500-1000	25	25.0
Less than 500	62	62.0
More than 2000	2	2.0
Total	100	100.0
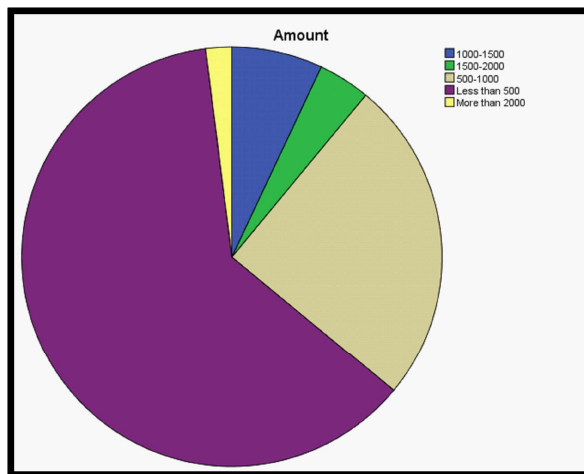
Fig.1. Distribution of average amount spent by customers per visit to a fast food restaurant in urban Maharashtra.

It is observed that the majority of consumers spend less than Rs.500 per visit. India is a developing country. Even though there has been increase in consumption of fast food, the purchasing power of consumers has seen little increase over the years. This fact is confirmed from the data collected.

We test the following hypothesis using Fisher's Exact test:

H_{01}: Gender of Respondent and Amount Spent are independent.

H_{11}: Gender of Respondent and Amount Spent are not independent.

> Fisher's Exact Test for Count Data
> data: Gender*Amount
> p-value = 0.006061
> alternative hypothesis: two.sided

Since, p-value < 0.05, we reject the null hypothesis H_0. Thus, we can conclude from the data collected, that gender of respondent and amount spent are dependent on each other.

H_{02}: Economic Status and Amount Spent are independent.

H_{12}: Economic Status and Amount Spent are not independent.

> Fisher's Exact Test for Count Data
> data: Economic Status*Amount
> p-value = 0.5112
> alternative hypothesis: two.sided

Since, p-value > 0.05, we do not reject the null hypothesis H_0. Thus, we can conclude from the data collected, that economic status and amount spent are independent of each other.

H_{03}: Occupation of Respondent and Amount Spent are independent.

H_{13}: Occupation of Respondent and Amount Spent are not independent.

> Fisher's Exact Test for Count Data
> data: Occupation*Amount
> p-value = 0.002486
> alternative hypothesis: two.sided

Since, p-value < 0.05, we reject the null hypothesis H_0. Thus, we can conclude from the data collected, that occupation of respondent and amount spent are dependent of each other.

We perform factor analysis to extract the significant factors from the data.

Kaiser-Meyer-Olkin factor adequacy

Overall MSA = 0.84

MSA for each item =

Food Taste and Quality.	Menu.Variety
0.87	0.84

Quick and Prompt Service	Ambience.. Decor. and. Environment
0.89	0.79

Restaurant. Popularity	Affordability
0.82	0.83

Discounts Offers	Location and Accessibility
0.78	0.90

Privacy

0.78

$chisq

273.1627

$p.value

3.099426e-38

$df

36

KMO>0.5 indicates that the data is appropriate for factor analysis. The p-value for Bartlett test is very small, suggesting that correlation matrix is different from identity matrix. We obtain the number of factors to extract using eigenvalue criterion and scree plot.
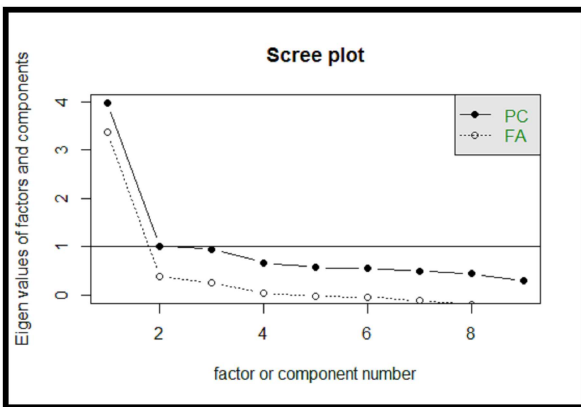
Fig.2. Scree plot of eigenvalues vs factor component number based on collected data.

From above scree plot, using principal component method, we can extract two significant factors. These two significant factors can be labeled as '**Service &**

Atmosphere' and '**Value for Money**' based on factor loadings tabulated below.

Factor Loadings

	Value for Money	Service & Atmosphere
Food Taste and Quality	0.77	0.13
Menu Variety	0.62	0.25
Quick and Prompt Service	0.47	0.57
Ambience, Décor and Environment	0.18	0.84
Restaurant Popularity	0.45	0.61
Affordability	0.76	0.14
Discounts/Offers	0.47	0.28
Location and Accessibility	0.70	0.26
Privacy	0.12	0.78

Since, it is observed from the data that consumers eat less often in fast food restaurants and spend less money per visit, the customer expects to get totally satisfied with his/her meal at the restaurant in the amount he pays and hence, '**Value for Money**' becomes an important criterion for urban Maharashtrian diners while choosing a particular fast food restaurant. Similarly, the customer also wishes to receive proper service and have a pleasant experience and hence '**Service and Atmosphere**' plays a key role for such consumers.

Conclusion

From the study, we have derived the following results:

- Factor analysis indicate that **Value for Money** and **Service & Atmosphere** are the two key factors customers take into account while choosing a particular fast food outlet to dine at. As customers tend to cautiously spend their money while dining out, they expect to get their money's worth when visiting a restaurant. Hence, it is advisable to fast food restaurants in urban Maharashtra to take these factors into consideration in order to increase their revenue.

- Descriptive statistics suggest that **Location and Accessibility**, **Restaurant Popularity** and **Food**

Taste and Quality affect customer's decision to visit a particular fast food restaurant.

• Gender and Occupation influence the average amount spent by a customer at a fast food restaurant.

• Customers prefer eating out with their friends, usually in the evenings. McDonald's is observed to be the most preferred fast food brand among young consumers, which may imply that burgers are the most popular food item among these consumers.

References

• Anderson, E. W., Sullivan M. W. (1993), The antecedents and consequents of customer satisfaction for firm, Marketing Science 12(2), 125-143.

• Chitnis S. (2019), A study on scenario of fast food industry in India, International Journal on Trend in Scientific Research and Development, Special Issue.

• Goyal A., Singh N., (2007) Consumer perception about fast food in India: an exploratory study, British Food Journal, Volume 109 No.2.

• Kara A., Kaynak E., Kucukemiroglu O. (1995), Marketing strategies for fast food restaurants: a customer view, International Journal of Contemporary Hospitality Management, Volume 7 No. 4

• Mahmud J. (2018), Relationship between demographics and consumer behaviour – A study on consumption of restaurant food by tertiary level students in Dhaka City, IOSR Journal of Business and Management, Volume 20, Issue 5, Ver. VII.

• Njelita C I, Anyasor O M (2020), Customer loyalty and patronage of quick service restaurant in Nigeria, British Journal of Marketing Studies, Volume 8, Issue 2.

• Olise M., Okoli M., Ekeke J., (2015), Factors influencing customer patronage of fast food restaurants: A study of selected customers of fast food in Anambra State, Nigeria, International Journal of Economics, Commerce and Management, Volume III, Issue 11.

• Y Prabhavathi, N T Krishna Kishore, M. Ramesh Kumar (2014), Consumer preference and spending pattern in India fast food industry, International Journal of Scientific and Research Publications, Volume 4, Issue 2.

A Study on Effects of Caffeine Consumption on College Students

Kashish Sutaria

S.Y.B.Sc Applied Statistics & Data Analytics

SVKM's Mithibai College of Arts, Chauhan Institutes of Science & Amrutben Jivanlal College of Commerce and Economics, Vile Parle.

Divya V Nair Assistant Professor in Biostatistics

Topiwala National Medical College and B.Y.L Nair Ch Hospital, Mumbai Central.

Abstract : Caffeine has been gaining popularity gradually over the years among college/university students. It is widely known for its stimulant properties which is beneficial to students in the short-term.

Aim : To study and analyse effect of caffeine consumption among college students in the short-term.

Methods : A random sample of 234 students was calculated using the sample size formula given by Cochran with proportion 0.813 cited in previously studied Research Paper on Study of Caffeine Consumption Patterns and Dependence among Management and Science University Students (7).

The data was collected through online questionnaire sent among college students ranging from Junior College to Post graduation students.

Findings: Out of 234 students, 197 (84.2%) of the respondents consumed caffeine whereas 37 (15.8%) did not consume caffeine. Of which, 53% consume caffeine as it gives them an energy boost whereas 48.2% consume caffeine as it relaxes them.

Keywords : *Caffeine, Consumption, effect, Cochran.*

1. Introduction

Caffeine is the most ubiquitous stimulants used worldwide. It is a naturally occurring substance of plant origin. According to a study published by the National Library of Medicine, 92% of college students consume caffeine in the form of coffee. The average caffeine intake per day is approximately 128.8mg in Korea and between 210 and 238 mg for Americans through various types of food including caffeinated beverages (2). Additionally 89% of adults in the U.S drinks caffeinated beverages, of which 64% drinks coffee, 18% drinks soft drinks and 16% drinks tea and the same pattern is followed with Koreans (3).

The survey on effect of Caffeine consumption on the health of college students were done among Northern Indiana which reported that "I drink caffeinated beverages because I like the taste" received the highest agreement. It was also observed that even though college students are aware about the consequences of negative impact of caffeine still they consumed as they are addicted to it (4).

Coffee consumption amounted to 1,21,060 kilogram bags across India during financial year 2022, according to Statistical Research Department. Caffeine present in various drinks and substances such as coffee, green tea, energy drinks etc, is used by people all over the world for a plethora of purposes, the most abundant being "wanting to feel awake". It is not lesser-known knowledge that short term effects of caffeine are beneficial to its consumers such as increased heart rate, increased alertness, increased physical energy etc. Due to its addictive nature and useful effects on such short notice, caffeine has proved itself to be the go-to stimulant for teenagers and students. Caffeine consumption is prevalent among college students as they feel that it boosts their energy. It relaxes to their bodies and rejuvenates their minds. (5). It is also observed that smoking influences the consumption of caffeinated beverages (6). It is nowadays observed that many products mention the presence of caffeine in their products. Nowadays, caffeinated products have labels consisting of its advantages and disadvantages. Even though the impact of caffeine on health is widely known, very few studies described the reasons behind the over-use of caffeine, thus it is necessary to understand the reasons behind consuming caffeinated products. Consuming caffeine without understanding its pros and cons creates dangerous situations especially among college students. The purpose of this study was first, to understand the short-term and long-term effects of caffeine consumption on academics.

Second, to know about the development of habit towards consuming caffeine. Third, to understand the effect of caffeine on the health of students.

2. Materials and Methods

2.1 Participants

A survey was administered among college students from Mumbai. A sample of 234 was collected among these colleges using Simple Random Sampling Techniques. The sample size was calculated using the information from previous study i.e. almost 81.3% students are likely to consume caffeine (7). The formula for calculating sample size is given by Cochran defined as follows:

$$n = \frac{Z_\alpha^2 p\, q}{d^2}$$

; where α=Level of significance=0.05*

Zα =Normal table value when α = 0.05 = 1.96

p=proportion of students consuming caffeine using previous study=0.813

q=1–p=0.187

d=margin of error=0.05*

*Assumed 5% level of significance and 5% margin of error.

Hence the resulted outcome for sample size is recorded as n = 233.6169 *H"* 234. The purpose of the survey was clearly explained to the study participants and hence the survey was conducted. The questionnaire was created through Microsoft forms and sent to every participant included in the study. A total of 251 responses were recorded, of which the data was cleaned by removing all outliers and incomplete responses. Hence, a total of 234 remained and were included in the analysis.

2.2 Measures

The participants were asked about their reasons of consuming caffeine. All questions were close ended. The amount of caffeine they consume per day was clearly asked to each participant (in cups). The questions were present with 4-point (1 –Never, 4 – Always) and 5-point Likert scale (1 –Strongly disagree, 5 – Strongly agree). The questions were asked based on their habits and preference to consume caffeinated products and their attitude and perception towards consuming caffeine. The demographic information of respondents was also collected.

1.3 Data Analysis

Data was coded using Excel and analyzed with Microsoft Excel and Statistical Package for the Social Sciences (SPSS version 23, IBM Corp, 2018). A descriptive analysis was performed on the demographic characteristics of college students. Suitable bar diagrams and pie diagrams are used to represent the percentages for qualitative data. A Chi square test is used to understand the association between variables. Further, Mean and Standard Deviations are used to analyse the quantitative data. The correlation analysis is used to understand the relationship between variables.

3. Results

3.1 Demographic and Caffeinated Beverage Consumption Characteristics.

From a random sample of 234 participants, proportion of caffeine consumers (p) was 0.84 and 0.16 (q) were non consumers. 42% respondents were male, 55% were female and 0.017% preferred not to say their gender. Out of the caffeine consumers 55.84% were females, 42.13% were males and 0.02% preferred not to say. The following TableNo.1 shows mean age of the respondents. The age range of respondents is 16 - 30 years. Mean age of respondents who do and do not consume caffeine is 20 years with a standard deviation of 2.6165.

Table No.1

Descriptive Statistics					
	N	Minimum	Maximum	Mean	Std. Deviation
Age (in years)	234	16.0	30.0	20.111	2.6165

Graph No.1

Cups of coffee,tea or other forms of caffeine consumed in a day

	2 cups or less	3 - 4 cups	5 - 6 cups	7 - 8 cups	9 cups or more
■ Total	164	25	6	1	1
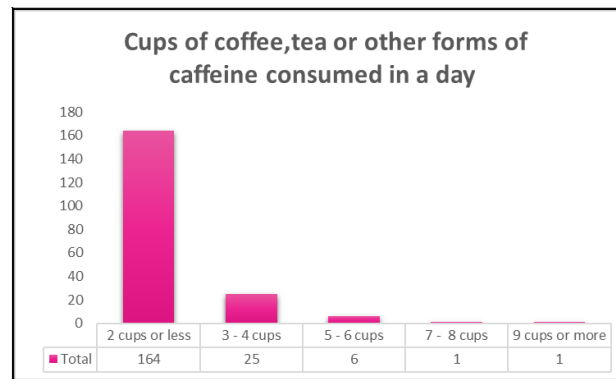
The above Graph No.1 shows that 164 out of 197 caffeine drinkers i.e. 83.25% drink 2 cups or less/day whereas 25 out of 197 i.e.12.69% drink 3-4 cups a day

3.2 Reasons behind using caffeinated products.

Graph No .2

Consuming Caffeine gives an energy boost
10% — Strongly agree
44% — Agree
37% — Neutral
6% — Disagree
3% — Strongly disagree

Consuming caffeine is relaxing
8% — Strongly agree
40% — Agree
33% — Neutral
12% — Disagree
7% — Strongly disagree

Caffeine helps to stay awake and alert
14% — Strongly agree
43% — Agree
17% — Neutral
18% — Disagree
8% — Strongly disagree

Caffeine helps to study late at night
18% — Strongly agree
43% — Agree
24% — Neutral
6% — Disagree
9% — Strongly disagree

Caffeine increases study hours
12% — Strongly agree
38% — Agree
26% — Neutral
13% — Disagree
11% — Strongly disagree

As seen in the above Graph No 2, Caffeine is mainly consumed by respondents to get an energy boost, to not feel drowsy or to stay alert and awake, to feel relaxed, to study late at night and for the taste. Of which, 44% of respondents agree and 10% strongly agree that caffeine gives the man energy boost.

National Conference on Applied Mathematics and Statistics (NCAMS-2023)

3.3 Habit of consuming caffeinated products.

The students participated in this survey have a habit of consuming less than or equal to 2 cups of caffeinated drinks. It is also observed that 12.7% consume 3-4 cups a day. They consume caffeine in copious amounts during examination week to stay them awake. This habit of drinking caffeine is due to their exam schedules.

3.4 Effect of caffeine consumption on health and academics

Using Correlation method, we observe the type of correlation between consuming caffeine during exam week and its stimulant effect of helping to stay awake. We get correlation value of 0.439501 which is a weak positive correlation. We can say that consuming caffeine during examination week does help one to stay awake but it may not be the only reason. The reasons may also be fear of failure, stress etc.

We used Chi-square test with Null hypothesis (Ho) that there is no association between given variables against Alternative hypothesis (H1) that there may be some association between given variables at 5% level of significance.

Table No: 2

Consumption of caffeine		Caffeine increases self-confidence.					Total
		Agree	Disagree	Neutral	Strongly agree	Strongly disagree	
During exami- nation week?	**Always**	16	9	11	9	4	49
	Never	1	15	9	0	15	40
	Often	10	22	19	0	5	56
	Rarely	6	19	17	0	10	52
Total		33	65	56	9	34	197

Table No: 2.1

Chi-Square Tests				
	Value	df	P value	Association is
Pearson Chi-Square (with continuity correction)	60.860	12	.000	Significant

TableNo : 3

Consumption of caffeine		Caffeine increases self-confidence.					Total
		Agree	Disagree	Neutral	Strongly agree	Strongly disagree	
During exami- nation week?	**Always**	20	1	3	24	1	49
	Never	11	5	10	2	12	40
	Often	31	3	16	5	1	56
	Rarely	23	4	18	4	3	52
Total		85	13	47	35	17	197

Table No: 3.1

Chi-Square Tests				
	Value	df	P value	Association is
Pearson Chi-Square (with continuity correction)	80.824	12	.000	Significant

Table No: 4

Consumption of caffeine		Caffeine increases self-confidence.					Total
		Agree	Disagree	Neutral	Strongly agree	Strongly disagree	
When	Always	21	1	4	11	1	38
you	Never	18	11	26	1	8	64
stressed?	Often	22	3	13	2	0	40
	Rarely	18	8	23	2	4	55
Total		79	23	66	16	13	197

Table No: 4.1

Chi-Square Tests				
	Value	df	P value	Association is
Pearson Chi-Square (with continuity correction)	53.330	12	.000	Significant

The above Table No 2.1, 3.1 and 4.1 show that there is association between consuming caffeine during examination week and increased self-confidence as well as increased overnight study. There is also association between consuming caffeine when stressed and feeling relaxed after consuming it. We find similar results when we check association between having some form of caffeine during examination week and improved group work activity, getting an energy boost, staying awake till late at night, increased academic performance, increased study hours, and increased recalling power with a p value < 0.0001 for each variable resulting in a significant association.

4. Discussions

The aim of this study was to truly understand the effect of caffeine on the health of college students. Their positive and negative perception towards consuming caffeinated beverages was analysed. In the present study almost 84.2% of students consumed caffeine on daily basis. This value was similar to studies performed in Management and science University students in Malaysia (7). This value was also similar to the studies performed in University and Netherlands (1).When comparing the results between the previous study by (7) and this present study, it proves that almost every student consumes caffeine frequently and the reasons are almost similar with regard to their beliefs. In this study almost 96.2% students know that coffee is the most caffeinated among the ones mentioned. Students believe that consuming 2 cups or less caffeine

in a day increases their study hours. Almost 36.32% of students feel that consuming caffeinated beverages helps them with overnight study. There are almost 37 students who do not consume caffeinated drinks as they believe it is not good for their health and so me even dislike it due to the taste and smell. Reports from Australia and Korea showed that caffeine consumption and energy drink intake was correlated with the poor quality of sleep among adults (8). In contrast, other reports showed no correlation between caffeine consumption and poor quality of sleep (9). Like previous studies (10) regarding sleep quality, factors that could affect sleep were not considered in the present study. Furthermore, specific socio-demographic variable like socio-economic status was not assessed. The hazardous factors that affect their health due to consuming caffeine were not assessed. The data collection was done during exams of many students which might have affected the answers given by the students. It could be expected that patterns determined in the present study may change in more demanding academic periods. It is seen that after having the awareness of negative impact of consuming caffeine, students prefer to drink caffeine just to get relief and relaxation. The study results that there is lack of actual or enough awareness about negative impact of caffeine on students' health. Awareness camps or any other measures such as programmes or talks in schools and colleges, should be taken to make students more aware about the dangers and adverse effects of excessive caffeine consumption. Parents

National Conference on Applied Mathematics and Statistics (NCAMS-2023)

and guardians should be equally educated and have adequate knowledge of long-term and short-term effects of consuming caffeine.

5. Limitations

The main limitation of the study is that it is survey-based study which can affect the accuracy of reported details. Many students may have not responded to the study truthfully even after giving consent. Another limitation is, it is nearly impossible to estimate the amount of caffeine consumed by the students and many factors such as health, sleep quality, life style etc has not been considered. Also, coffee and some energy drinks contain more caffeine than tea. The difference in level of caffeine in different substances has not been considered.

6. Conclusions

We conclude that majority of college students (84.2%) consume caffeine in one form or another for various reasons which may have adverse effects on their health in the long run. The top reasons for consuming caffeine were:

Getting an energy boost

Feeling relaxed (normally and when stressed)

Increasing study hours

Improved group work activity

Increasing recalling power

Liking the taste

Increased academic performance

Helping to stay awake till late at night

Most of the above stated associations were made with consumption of caffeine during examinations. One can assume that students consume more than the recommended or even permissible amount of caffeine during exams which may lead to short-term effects which are beneficial such as increased self-confidence, increased study hours etc. However, this has its cons as well such as disrupted sleep cycle, headaches, nausea etc. as caffeine is known for its stimulant properties. In the current study, 36.04% respondents said that they cannot fall asleep easily if they have consumed caffeine during the day. Many may feel its withdrawal symptoms as well which may lead to consuming it repeatedly which may ultimately result in an addiction. Thus, we can say that caffeine indeed has beneficial properties and effects but only when consumed within the recommended dose. It has favourable short-term effects on academics and health by increasing performance, alertness , energy, and study hours. Long-term effects of caffeine are difficult to isolate as there may be other factors involved such as metabolism, sleep quality, physical activity etc.

7. References

- Riera-Sampol A, Rodas L, Martínez S, Moir HJ, Tauler P. Caffeine Intake among Undergraduate Students: Sex Differences, Sources, Motivations, and Associations with Smoking Status and Self-Reported Sleep Quality. Nutrients. 2022 Apr16; 14(8):1661.

- Krieger D, Kalman D, Feldman S, Arnillas L, Goldberg D, Gisbert O, etal. The Safety, Pharmacokinetics, and Nervous System Effects of Two Natural Sources of Caffeine in Healthy Adult Males. Clin Transl Sci. 2016 Oct; 9(5) : 246–51.

- Lee H, Park H. Testing the Impact of Message Interactivity on Relationship Management and Organizational Reputation. J Public Relat Res. 2013 Apr ; 25(2):188–206.

- Bucher J, Fitzpatrick D, Swanson AG, Abraham SP. Caffeine Intake Habits and the Perception of Its Effects on Health Among College Students. Health Care Manag.2019 Jan ; 38(1):44–9.

- Wise G.E., Santander D. Assessing the History of Safe Use of Guayusa J. Food Nutr Res. 2018 Aug12; 6(7) : 471–5.

- Choi J. Motivations Influencing Caffeine Consumption Behaviors among College Students in Korea: Associations with Sleep Quality. Nutrients. 2020 Mar 30; 12 (4):953.

- Atikah Ramli NA, Sriperumbuduru VPK, Ghazi HF, Dalayi N J. A Study of Caffeine Consumption Patterns and Dependence among Management and Science University Students. Indian J Forensic Med Toxicol.2019; 13 (1):101.

52

- Watson E, Coates A, Kohler M, Banks S. Caffeine Consumption and Sleep Quality in Australian Adults. Nutrients. 2016 Aug 4; 8 (8):479.

- C. Janson, T. Gislason, W. De Backer etal., "Prevalence of sleep disturbances among young adults in three European countries," Sleep, vol.18, no.7, pp.589–597, 1995.

- Kharaba Z, Sammani N, Ashour S, Ghemrawi R, Al Meslamani AZ, Al-Azayzih A, etal. Caffeine Consumption among Various University Students in the UAE, Exploring the Frequencies, Different Sources and Reporting Adverse Effects and Withdrawal Symptoms. Valdés-RamosR,editor. JNutrMetab.2022May18;2022:1–7.

Excess Specific Acoustic Impedence, for Binary Mixtures of Vinyl Acetate + Ethers

Dr. Ajit N. Bhumkar

Physical Chemistry Research Laboratory, B .N. Bandodkar College of Science, Thane- 400601, INDIA

Abstract : Densities (ρ), viscosities (η) and speeds of sound (u) of binary mixtures of vinyl acetate with ethers (butyl vinyl ether, diisopropyl ether, anisole and dibutyl ether) at (303.15,308.15 and 313.15) K were measured over the entire composition range. From these experimental data, excess specific acoustic impedence Z^E were calculated. These quantities have been fitted to the Redlich-Kister polynomial equation and results analysed in terms of molecular interactions and structural effects. The obtained data for viscosity were fitted to Mc-Allister (4-body) viscosity model.

Introduction

Thermodynamics plays an important role in numerous industries in the design of separation equipment and process as well as for product design and optimizing formulations. Complex polar and associating molecules are present in many applications for which different type of thermodynamic and transport properties need to be known over wide ranges of temperature and pressure. Vinyl acetate is a polar simple aliphatic ester that is full of benefits and find many industrial applications. It is a chemical intermediate used in various dyes. It can be used as a solvent for a number of synthetic and natural resins. It is used as cost effective additive based in the dye sensitized solar cells. Likewise ethers are used for many purposes and find good applications in chemical engineering areas. Due to fast changing trends of the environmental concerns the need for oxygenated compounds (ethers) are becoming important because of diminishing petroleum reserves and increasing air pollution. The oxygenated fuels are mostly octane enhancers and reduce carbon monoxide emission.

Experimental

All chemicals utilized in this study were of analytical grade supplied by Sigma-Aldrich. The binary liquid mixtures were prepared by mixing known masses of pure liquids in airtight stoppered bottles. Densities of pure and their binary mixtures were determined using a density meter (DDM -2910 Rudolph Research Analytical). Viscosities of the pure liquids and their mixtures were determined by using Ubbelohde viscometer. The speeds of sound of pure liquids and liquid mixtures were determined bsy using a single-crystal variable path interferometer (model F-81) supplied by Mittal Enterprises, New Delhi, India operating at frequency of 2 MHz.

Result and Discussion

The variation in excess specific acoustic impedence Z^E Vs mole fraction (x_1) of vinyl acetate are plotted in Figure . It is observed that for the system anisole the Z^E is completely positive over the entire composition range while for all the remaining systems (butyl vinyl ether, diisopropyl ether and dibutyl ether) the Z^E values bare completely negative. The system diisopropyl ether exhibit large negative values while for dibutyl ether these values are less negative. The negative values of Z^E at the studied temperatures for (Diisopropyl ether, dibutyl ether and butyl vinyl ether) suggest the presence of weak interactions between the component molecules of the mixtures.
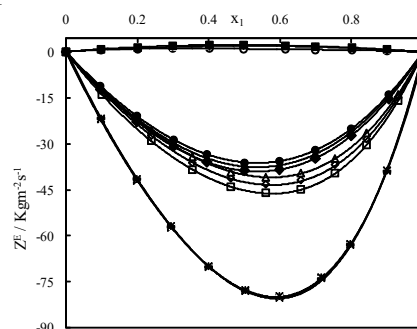
Fig. Curves of excess acoustic impedence(Z^E)Vs mole fraction(x_1) for the binary mixtures of,

Vinyl acetate+ Butyl Vinyl ether at (□, 303.15; ◊, 308.15; Δ, 313.15) K,

Vinyl acetate+Diisopropyl ether at (×,303.15; Ж, 308.15 ; −, 313.15) K,

Vinyl acetate+Anisole at (O, 303.15;+, 308.15; ■, 313.15) K,

Vinyl acetate+ DiButyl ether (♦, 303.15; ▲, 308.15 ; ●, 313.15) K

References

[1] Jaime Wisniak , Rene´ D. Peralta , Ramiro Infante, Gladis Cortez Densities and derived thermodynamic properties of the binary systems of 1,1- dimethylethyl methyl ether with allyl methacrylate, butyl methacrylate, methacrylic acid, and vinyl acetate at T = (298.15 and 308.15) K, J. Chem. Thermodynamics, 37 (2005) 729–736

[2] Hamid Reza Raee, Farshid Frouzesh, Shokofeh Miri Volumetric properties for binary mixtures of ethyl acetate, vinyl acetate and tert-butyl acetate with 1-propanol and iso-butanol at T=(293.15–313.15) K and P=0.087 MPa, Journal of Molecular Liquids 213 (2016)255-267

[3] L. Grunberg, A.H. Nissan, Mixture Law for Viscosity, Nature. 164 (1949) 799-800.

[4] V. M. Lobe, M.S. Thesis, University of Rochester, Rochester, New York (1973).

[5] M.R.V. Krishnan, G.S. Laddha, Viscosities of Binary Liquid Mixtures, Prediction from Y-X Data, Ind. Eng. Chem. Fund. 7 (2) (1968) 324-327.

[6] O. Redlich, A. T. Kister, Algebraic representation of thermodynamic properties and the classification of solutions, Ind. Eng. Chem. 40 (1948) 345-348.

Hasse-Minkowski principle for 2×2 matrices over algebraic number fields

Tanay Kochrekar[*]   A. S. Garge[†]

Department of Mathematics, University of Mumbai, Vidyanagari, Santacruz East, Mumbai, Maharashtra, 400098, India.

Abstract

In this paper, we study the Hasse-Minkowski theorem for matrix quadratic forms over algebraic number fields. We prove the local global principle for matrix quadratic forms for 2×2 matrices over algebraic number fields.

Mathematics Subject Classification 2020: 13F30, 11D88, 11E08, 11E12, 15A63, 11F85.

Key words: Algebraic number fields, p-adic valuations, Completions of fields, Hasse-Minkowski theorem, Quadratic form, Isotropy

1 Introduction

Throughout this paper, K will be an algebraic number field. Recall that an algebraic number field is by definition, a field extension of $\mathbb{Q}$ of finite degree. A valuation v of K that we consider will not be an extension of usual 2-adic valuation on $\mathbb{Q}$. A quadratic form in n variables is a homogeneous polynomial f of degree two in the variables $X_1, X_2, \ldots, X_n$, given by:

$$f\left(X_1, X_2, \ldots, X_n = \right) \sum_{1 \leq i \leq n} a_i X_i^2 + \sum_{1 \leq i < j \leq n} 2a_i a_j X_i X_j.$$

An isotropy of a quadratic form over K is a non-zero vector, $(x_1, x_2, \ldots, x_n) \in K^n$, such that $f(x_1, x_2, \ldots, x_n) = 0$. We say that a quadratic form f over K is isotropic, if it has an isotropy.

---

[*]tanaykochrekar@gmail.com
[†]anuradha.garge@gmail.com

For example, the quadratic form $X_1^2 - X_2^2$ (called the hyperbolic plane) is obviously isotropic, but the form $X_1^2 + X_2^2$ over the field $\mathbb{Q}$ is not isotropic.

The classical theorem which we wish to recall here is due to Hasse-Minkowski, which gives a necessary and sufficient condition for a quadratic form over such a field to be isotropic in terms of local conditions.

Let K be any field. A valuation of K is a map $v : K \to \mathbb{R}^+$ ($,\mathbb{R}^+$ denoting the non-negative real numbers), such that for $x, y \in K$,

1. $v(x) = 0$ if and only if $x = 0$.

2. $v(x + y) \leq v(x+)\, v(y)$

3. $v(xy=)\, v(x)v(y)$ for $x, y \in K$.

The map $v : K \to \mathbb{R}^+$ defined by $v(0) = 0$ and $v(x) = 1$ for $x \neq 0$ is obviously a valuation of K called trivial valuation. From now on by a valuation we mean a non-trivial valuation. We have a notion of equivalence of valuations defined by: $v \sim v'$, if and only if there exists a positive real number c such that $v' = v^c$. It is easy to see that v and v' are equivalent if and only if for each $x \in K, v(x) < 1$ implies $v'(x) < 1$. For any valuation v of K, we denote by K_v, the completion of K, with respect to $v : K_v$ is in fact the completion of K with respect to the metric defined for $x, y \in K$ by $d(x, y=)\, v(x - y)$. The K_v has an obvious field structure and K is embedded inside K_v as a dense subfield.

We note a theorem by Ostrowski:

Theorem 1. [2], [Theorem 2.1.2] *Every non-trivial absolute value on $\mathbb{Q}$ is equivalent to one of the absolute values $|\ |_p$, where p is a prime number or $p = \infty$. $\left|\frac{a}{b}\right|_p = v_p(a) - v_p(b)$, where $v_p(n)$ is the highest power of p which divides n.*

Instead of dealing with general quadratic forms, one can restrict to diagonal ones over certain fields using the lemma below:

Lemma 1. [1], [Proposition 3] *Any quadratic form (over a field of characteristic different from 2), is equivalent to a diagonal form.*

The Hasse-Minkowski theorem is given by:

Theorem 2. [1], [Theorem 1] *Let f be a quadratic form over an algebraic number field K. If f is isotropic over K_v for all valuations v, then f is isotropic over K.*

2 Hasse-Minkowski theorem for $2{\times}2$ matrices

We wish to establish the matrix version of the above theorem for 2×2 matrices. For this we first give some definitions.

Let K be a field with characteristic different from two. A diagonal quadratic form in m matrix variables over $M_2(K)$ is a homogeneous polynomial of degree 2 given by

$$f(X_1, \ldots, X_m{=}\,)\, a_1 X_1^2 + \cdots + a_m X_m^2.$$

By Hasse-Minkowski theorem of the matrices, we mean the following:

Theorem 3. *The matrix diagonal quadratic form,* $\displaystyle\sum_{i=1}^{m} a_i X_i^2 = 0$ *has non trivial solution over* $M_2(K)$ *if and only if it has a non-trivial solution in* $M_2(K_v)$ *for every valuation* v. *(Here, by non-trivial solutions, we mean a set of matrices* $X_i \in M_2(K)$, *not all zero, such that* $\displaystyle\sum_{i=1}^{m} a_i X_i^2 = 0$. *Note that, it may happen that* $X_i \neq 0$, *but* $X_i^2 = 0$ *for* $1 \leq i \leq m$.)

Proof. Suppose that the diagonal quadratic matrix form has a non-trivial solution over K. The embedding of K in K_v gives the required non-trivial solution over $M_2(K_v)$ for each valuation v.

We now prove the converse i.e. we assume that the matrix diagonal quadratic form, $\displaystyle\sum_{i=1}^{m} a_i X_i^2 = 0$ has non trivial solution in $M_2(K_v)$ for every valuation v.

$f(X_1, \ldots, X_m{=}\,)\, a_1 X_1^2 + a_2 X_2^2 + \cdots + a_m X_m^2.$

Let $X_k = \begin{bmatrix} r_k & s_k \\ t_k & u_k \end{bmatrix}$ for $1 \leq k \leq m$.

$f(X_1, \ldots, X_m{=}\,)\, \displaystyle\sum_{k=1}^{m} a_k \begin{bmatrix} r_k^2 + s_k t_k & r_k s_k + s_k u_k \\ t_k r_k + u_k t_k & t_k s_k + u_k^2 \end{bmatrix}.$

Essentially, we have to find a simultaneous solution of the system of equations

$$\sum_{k=1}^{m} a_k(r_k^2 + s_k t_k) = 0. \tag{1}$$

$$\sum_{k=1}^{m} a_k(r_k s_k + s_k u_k) = 0. \tag{2}$$

$$\sum_{k=1}^{m} a_k(t_k r_k + u_k t_k) = 0. \tag{3}$$

$$\sum_{k=1}^{m} a_k(t_k s_k + u_k^2) = 0. \tag{4}$$

Firstly, we set $r_k = -u_k$. This makes the non-diagonal entries of all of the X_i^2 matrices zero. Now, we need to only consider the cases where $r_i^2 + s_i t_i \neq 0$ for at least one of the i, $1 \leq i \leq m$.

These equations can be modified to

$$\sum_{k=1}^{m} a_k(r_k^2+) \sum_{k=1}^{m} \frac{a_k}{4}(s_k + t_k)^2 - \sum_{k=1}^{m} \frac{a_k}{4}(s_k - t_k)^2 = 0. \tag{5}$$

$$\sum_{k=1}^{m} a_k s_k(r_k + u_k) = 0. \tag{6}$$

$$\sum_{k=1}^{m} a_k t_k(r_k + u_k) = 0. \tag{7}$$

$$\sum_{k=1}^{m} a_k(u_k^2+) \sum_{k=1}^{m} \frac{a_k}{4}((s_k + t_k)^2 - (s_k - t_k)^2) = 0. \tag{8}$$

The expression on the left hand side of equation (5) above,

$$\sum_{k=1}^{m} a_k(r_k^2+) \sum_{k=1}^{m} \frac{a_k}{4}(s_k + t_k)^2 - \sum_{k=1}^{m} \frac{a_k}{4}(s_k - t_k)^2$$

can be seen as a quadratic form over K_v.

It can be made into a diagonal quadratic form equation:

$$\sum_{k=1}^{m} a_k R_k^2 + \sum_{k=1}^{3} c_k Y_k^2 + \sum_{k=1}^{m} d_k Z_k^2 = 0,$$

with the change of variable $R_k = r_k, s_k + t_k = Y_k, s_k - t_k = Z_k$, with new coefficients: $c_k = -d_k = \frac{a_k}{4}$. By assumption, the above diagonal quadratic form admits a solution over K_v for each v. Hence by Hasse-Minkowski theorem,

$$\sum_{k=1}^{m} a_k R_k^2 + \sum_{k=1}^{3} c_k Y_k^2 + \sum_{k=1}^{m} d_k Z_k^2 = 0$$

admits a non-trivial solution over K.

We observe that, if we substitute back the variables from (5), we get $s_k + t_k = y_k$ and $s_k - t_k = z_k$, for all $1 \le k \le m$. Thus $s_k = \frac{y_k + z_k}{2}$ and $t_k = \frac{y_k - z_k}{2}$, for all $1 \le k \le m$. Note that $y_k \neq 0$ implies both s_k and t_k are not simultaneously zero i.e. the diagonal quadratic form associated with the $(1,1)$-th entry is isotropic over K.

Let $\overline{r_k}, \overline{y_k}, \overline{z_k}$ be the solutions, not all zero, of the diagonal quadratic form associated with the $(1,1)$-th entry for the variables R_k, Y_k, Z_k respectively, for all $1 \le k \le m$.

Further we observe, since we set $\overline{u_k} = -\overline{r_k}$, the corresponding solutions for equation (8) are also obtained, where $\overline{u_k} = -\overline{r_k}$. The equations (6) and (7) are satisfied as well.

Now we will check that these non-trivial solutions work for the original system of equations, $(1), (2), (3), (4)$ as well. If they work for (1), then from the values chosen for $\overline{u_k} = -\overline{r_k}$, (8) is satisfied as well.

We know,

$$\sum_{k=1}^{m} a_k \overline{r_k}^2 + \sum_{k=1}^{m} c_k \overline{y_k}^2 + \sum_{k=1}^{m} d_k \overline{z_k}^2 = 0.$$

Substituting the original values back,

$$\sum_{k=1}^{m} a_k (\overline{r_k}^2 +) \sum_{k=1}^{m} \frac{a_k}{4} (\overline{s_k + t_k})^2 - \sum_{k=1}^{m} \frac{a_k}{4} (\overline{s_k - t_k})^2 = 0.$$

$$\sum_{k=1}^{m} a_k (\overline{r_k}^2 +) \sum_{k=1}^{m} \frac{a_k}{4} \left[(\overline{s_k + t_k})^2 - (\overline{s_k - t_k})^2 \right] = 0.$$

$$\sum_{k=1}^{m} a_k (\overline{r_k}^2 +) \sum_{k=1}^{m} \frac{a_k}{4} (4 \overline{s_k t_k}) = 0.$$

$$\sum_{k=1}^{m} a_k (\overline{r_k}^2 + \overline{s_k t_k}) = 0.$$

Hence the same values, $\overline{r_i}, \overline{s_i}, \overline{t_i}$ also are the solutions of (1).

Since we let, $\overline{u_k} = -\overline{r_k}$. We see that this assumption simultaneously makes all of the equations hold true.

At last, we see that not all of the X_k matrices are zero, therefore a non-trivial solution of matrices is obtained. $\qquad\square$

Now, we observe that such a solution exists for the matrices of any order, if it exists for 2×2 matrices.

Corollary 1. *Let K be an algebraic number field and let $n, m \geq 2$ be integers. The matrix diagonal quadratic form $\displaystyle\sum_{i=1}^{m} a_i X_i^2 = 0$ has non trivial solution over $M_n(K)$ if and only if it has a non-trivial solution in $M_2(K_v)$ for every valuation v.*

Proof. Since $\displaystyle\sum_{i=1}^{m} a_i X_i^2 = 0$ has a non-trivial solution in $M_2(K_v)$ for each v, it has a non-trivial solution in $M_2(K)$ by Theorem 1. Let $Y_k = \operatorname{diag}[X_k, 0_{n-2}.]$ Then, $\displaystyle\sum_{i=1}^{m} a_i Y_i^2 = 0$, and Y_i are non-trivial by construction, this gives the required non-trivial solution. $\qquad\square$

References

[1] R.Sridharan, On the theorem of Hasse-Minkowski, Cyclotomic Fields and Related Topics: Proceedings of the Summer School (June 7-30, 1999) Organized by Department of Mathematics, University of Pune, Bhaskaracharya Pratishthana.

[2] F. Gouvêa, p-adic numbers An introduction, Springer Universitext , Second edition, 1997.

PRE-CONFERENCE WORKSHOP

VPM'S B.N.Bandodkar College of Science (Autonomous),Thane

Department of Mathematics and Statistics
jointly organises

FIRST PRE - CONFERENCE WORKSHOP
OF
NATIONAL CONFERENCE ON
APPLIED MATHEMATICS AND STATISTICS (NCAMS-2023)

Our Guest

Topic:
How to write research paper in Mathematics??

Dr. Manisha Acharya
Vice- Principal Sc.Faculty and Head, Dept.of Mathematics, M.D.College

Scan it to register

Date: 26 November 2022
Time: 2:00 pm to 4:00 pm
Venue: Patanjali Auditorium

Organising Secretaries:
Ms.Minal Wankhede
Ms.Sumaiya Shaikh

Principal:
Dr.Moses Kolet

V.P.M's B.N.Bandodkar College of Science (Autonomous), Thane.

SECOND PRE - CONFERENCE WORKSHOP
OF
NATIONAL CONFERENCE ON
APPLIED MATHEMATICS AND STATISTICS (NCAMS-2023)

09:00 AM - 12:00 Noon
3 Dec, 2022
BRIMS, Panini Auditorium

Topics :
1. Writing a Research Paper in Statistics.
2. Use of LaTex in Research Paper.
3. Fuzzy Inventory Model.

Resource Persons :

Dr. Annapurna S.
Associate Professor,
Department of Statistics,
St. Xavier's College Autonomous

Mr. Mandar Bhanushe,
Head,
Faculty of Sci. and Technology,
IDOL, University of Mumbal

Ms. Tanvim Shaikh,
Assistant Professor,
Department of Statistics,
B.N.Bandodkar College of Science,(Autonomous)

Scan it to register :

Organizing Secretaries :
Ms. Minal Wankhede
Ms. Sumaiya Shaikh

Principal
Dr. Moses Kolet

First Pre-conference Workshop

On

How To Write Research Paper In Mathematics

First pre-conference workshop of National Conference on Applied Mathematics and Statistics to be held on 13th and 14th January 2023 by Department of Mathematics and Department of Statistics was organized on 26th November 2022 at 2pm at Patanjali Auditorium of B. N. Bandodkar College of Science (Autonomous), Thane. Guest Speaker Dr. Manisha Acharya, Vice-Principal and Head, Department of Mathematics, M. D. College gave a talk on "How to Write Research Paper in Mathematics".

Dr. Manisha Acharya explained step by step how to begin research in Mathematics. She explained in detail the process of writing a research paper. The example of Research Paper in Induction Principles of Mathematics given by her made students well acquainted with writing a paper in Mathematics. She threw light on ideas about different topics in which students can do research.

As the workshop was designed for undergraduate students, she also listed the role of a student as a volunteer in organizing a conference. At the end, she highlighted the benefits of attending the conference to students. She successfully created the enthusiasm among the students to write a research paper.

How to Write Research Paper in Mathematics

Dr. Manisha Acharya

Vice-Principal and Head,
Department of Mathematics, M. D. College

Research in Mathematics

- Decide the field of interest
- Algebra, Topology, Complex analysis, Graph Theory, Number Theory
- Decide Guide/ Supervisor / teacher
- Guide suggests some topic and also provides the material to read
- Search on Google
- Read articles on the relative topics from Journals.
- To study and analyze the existing research
- Make use of it to derive new results

Headings in a Research Paper

- Title
- Name of Authors
- Abstract
- Keywords
- Introduction
- Main Results
  - Result 1
  - Result 2
  - Result 3
- Illustration
- Applications
- References

Principles of Inductions

XYZ, B. N. Bandodkar College; ABC, B. N. Bandodkar College

Abstract : In this paper authors have defined few techniques and properties of Natural Numbers which can be used to check whether a certain rule or a formula exits for all natural numbers.

Key Words : Law of Trichotomy, well ordering principle, Induction Axiom

Introduction : In this paper we have shown that if a certain property is true for a natural number 1 and then further by assuming that the property is true for a certain natural number k if we can prove it for the next value ,that is , for (k+1) then that property is true for all natural numbers .

Definition, law of trichotomy , well ordering principle, notations, symbols etc in Introduction

Main Results

1. Induction Axiom
2. First Principle of Induction
3. Second Principle of Induction

Illustrations:

1.

2.

Applications

References

- Benefits of Attending the Conference
- Enhance the knowledge
- Discuss the doubts with an Author of the paper.
- Applications of Mathematics in other fields
- Interaction with Eminent Speakers who are stalwartsin the field
- Knowledgeable but simple
- Learn how to present yourself
- Develops the personality.

Role of Student as a Volunteer

- Letters and Mementoes for guests
- Folder / writing pad / Anyother Material
- Arrangement at different venues
- Arrangements for Refreshment

Second Pre-Conference Workshop

The Second pre- conference workshop of National Conference on 'Applied Mathematics and Statistics' was organised on 3rd December, 2022 from 9:00 am to 1:00 pm.

The Pre-Conference workshop was organised with an aim to enables students to prepare for the conference and to train them in their early stage of research, to carry out research activity using recent techniques in order to achieve high degree of productivity.

The second pre conference workshop had 3 sessions:

- Session I- Writing Research Paper in a Statistics by Dr. Annapurna S. from
- 9:30 am to 10:30 am
- Session II- Use of LaTex in a Research Paper Mr. Mandar Bhanushe from 10:40 am to 11:40 am
- Session III- Fuzzy Inventory Model by Ms. Tanzim Shaikh from 11:50 am to 1:00 pm.

Mr. Aniruddha Shelke and Mr. Rajeev Ghade students of T.Y.B.Sc. Statistics, compered the event. The event began with recite of Sarasvati Vandana by Amruta Thombhare student of S.Y.B.Sc (MS). Honorable Dr. Moses Kolet, Principal, while addressing the audience, emphasized the significance of research. Sir welcomed the participants and gave best wishes to the organizing committee for smooth conduct of the event. He also highlighted the importance of preparatory sessions and motivated students to participate in large number.

Mr. Manthan Mohite, student of T.Y.B.Sc. Statistics gave brief introduction of **Dr. Annapurna S.** The first session of workshop 'Writing Research Paper in a Statistics was taken by Dr. Annapurna S., Associate Professor, Department of Statistics, St. Xavier College Autonomous. She took very informative session covering all the aspects of writing Research Paper. Talk was followed by a question-answer session with the participants. This session was concluded by vote of thanks and concluding remarks from Mr. Manthan Mohite. Total 76 participants attended the workshop.

Mr. Ridhan More, student of T.Y.B.Sc. Statistics gave brief introduction of **Mr. Mandar Bhanushe.** The second session of workshop "Use of LaTex in a Research Paper' was taken by Mr. Mandar Bhanushe, Head, Faculty of Science and Technology, IDOL, University of Mumbai. He delivered lecture on how to create scientific documents that look professional and accurately reflect the precise equations and other graphics necessary to express the research work. Talk was followed by a question-answer session with the participants. This session was concluded by vote of thanks and concluding

remarks from Ms. Pradnya Barge student of T.Y.B.Sc. Mathematics. Total 51 participants attended the workshop.

Mr. Suraj Vishvakarma, student of T.Y.B.Sc. Statistics gave brief introduction of Ms. Tanzim Shaikh. The third session of workshop 'Fuzzy Inventory Model' was taken by **Ms. Tanzim Shaikh** , Assistant Professor, Department of Statistics, B.N. Bandodkar College of Science (Autonomous), Thane. She presented her research work and explained the standard format of writing a research paper. Talk was followed by a question-answer session with the participants. This session was concluded by vote of thanks and concluding remarks from Mr. Suraj Vishvakarma. Total 47 participants attended the workshop.

All the 3 sessions included multi-media resources to better elucidate the concepts.

Students understood that an original research articles like other scientific writing follows a standard format. The various sections are abstract, introduction, literature review methods, results, conclusion, acknowledgements, references.

Glimpse of the workshop :

Second Pre-Conference Workshop

1. Writing A Good Research Paper in Statistics

Dr. Annapurna Shankarnarayanan, PhD

Associate Professor, Department of Statistics
Vice Principal (Arts)
St. Xavier's College, Autonomous, Mumbai

This workshop was conducted for the undergraduate participants with the aim to ignite the spark of research. Hence before going into the core research aspect of the subject, it was imperative to explore and understand the fundamental meaning of research.

Broadly research offers a unique perspective of any topic under consideration. Research is crucial to any form of improvement. Academic research is an original work in written form reviewed by peers and published in journals of high repute which could be looked at as a culmination of an involved process of critical thinking, source evaluation, organization, composition and analysis.

In the domain of Statistics as also in many other natural sciences, research can be viewed in two aspects namely Theoretical research and Application based research.

Theoretical research are the building blocks that form the core of the subject. It develops the mathematical, computational, and philosophical foundations of Statistics. Its framework is built upon mathematical approach and conceptual thinking. It develops rigorous new statistical methods grounded in probability and statistical theory.

Applied research, many a times, has its roots firmly entrenched in Data Analysis. It focuses on applying statistical theories to solving real-world problems through data collection, processing, analysis and interpretation. It is applied in various fields such as medicine, information technology, engineering, finance, marketing, accounting, business, etc, the options being limitless.

A classic example of theoretical research is the birth of probability theory due to the questions posed by Chevalier de Méré Antoine in 1645 to Blaise Pascal and Pierre de Fermat regarding his loss in the gambling tables. The exchange of letters between Pascal and Fermat formulated the fundamental principles of probability theory, expected value, conditional probability etc. thus creating The Mathematical Theory of Probability.

An example of the application of Statistics to solve a real-world problem is the crash of Air France Flight 447 in the Atlantic Ocean flying from Rio de Janeiro to Paris on 1st June 2009 with 228 passengers and crew on board. Intense searching in three phases by Brazilian military, French nuclear submarine and two French-contracted ships did not help retrieve the black boxes, cockpit voice recorder, flight data recorder or the bodies (initially only 50 were recovered) as the crash area was not deciphered.

Finally in July 2010, a U.S. based search consultancy Metron, Inc. was engaged. Statisticians Stone and colleagues from Metron reexamined all evidences from previous search records, took into account prior information and used Bayesian inference to understand where the search should focus. Within a week of resuming search operations, a large portion of the debris-bodies, engines, wing parts discovered, aircraft's CVR and FDRs were found and 104 bodies were recovered from the wreckage. The entire methodology applied has been very well described in the research paper 'Search for the Wreckage of Air France Flight AF 4471' by Lawrence D. Stone, Colleen M. Keller, Thomas M. Kratzke and Johan P. Strumpfer. Thus the statement of a problem gives rise to a topic for research and creates the intellectual concern indicating why it is worthy of study. Then designing the research to clarify the problem, analysis of the data produced by the research follows and finally summary and conclusion of the study brings forth results.

All this needs to be penned commencing with a brief Title that summarizes the paper effectively. This is followed by an Abstract which is a concise, fully self-contained single paragraph that briefly outlines the entire paper with no reference to outside source. The

Abstract helps in submission to journals, selection as well as in indexing. Around 4 to 6 words that define the research field or topic are stated and are termed as Keywords. They help indexers and search engines find relevant papers, thus enabling readers to find work easily.

Research papers also state the alphanumerical classification scheme MSC2020

The main body of the paper will comprise of an Introduction giving the essence of what the paper offers in terms of research and is a crucial aspect of the paper. This would be followed by the Literature Search done by the author after which the Methodology and

Theoretical results such as theorems and proofs, characterizations, application of theorems and simulation studies are stated. The data collection with its interpretations and corresponding graphs and plots are stated in the paper after which at the end the references used are listed. The Conclusion prior to the References highlights the findings and provides a clear interpretation of the results and stresses the significance of the study

Thus, we surmise that good research in any discipline is about creating strong theories and applying the knowledge of this theory judiciously to a real-world phenomenon and deriving results that benefit society at large.

2. A Demo on LaTeX Software

Mr. Mandar Bhanushe

Head, Faculty of Science & Technology, IDOL, University of Mumbai

As a part of pre-conference workshop and seminars, on 3rd December 2022, a talk and demonstration session by Prof Mandar Bhanushe, was organised on "LaTeX: A mathematical typesetting software".

LaTeX falls in the category of FOSS i.e., free and open-source software. Most of the technical typing requirements like page numbering, sectioning, table of contents or table of figures, indexing, formatting, bibliography, cross-referencing etc are quite difficult and tiresome if done using MS Office software. Research articles or even thesis for that matter, runs into more number of pages, making it even more difficult for the authors to focus on the content and take care of the technical aspects of the content typing. LaTeX is not free but also quite logical and simple to use software. It is not a WYSIWYG kind of software. Anyone who is a professional, academician, researcher or even a student, interested in quality typing should look for LaTeX as a better alternative. It can compile big books consisting of thousands of pages and still the TeX file is only in few kbs only! LaTeX is also available in all operating systems like Windows, Mac and Unix! Here are some examples of LaTeX source and output files:

Order and numbering of the references are automatic. Inserting a new citation reorders the references (a cool feature !)
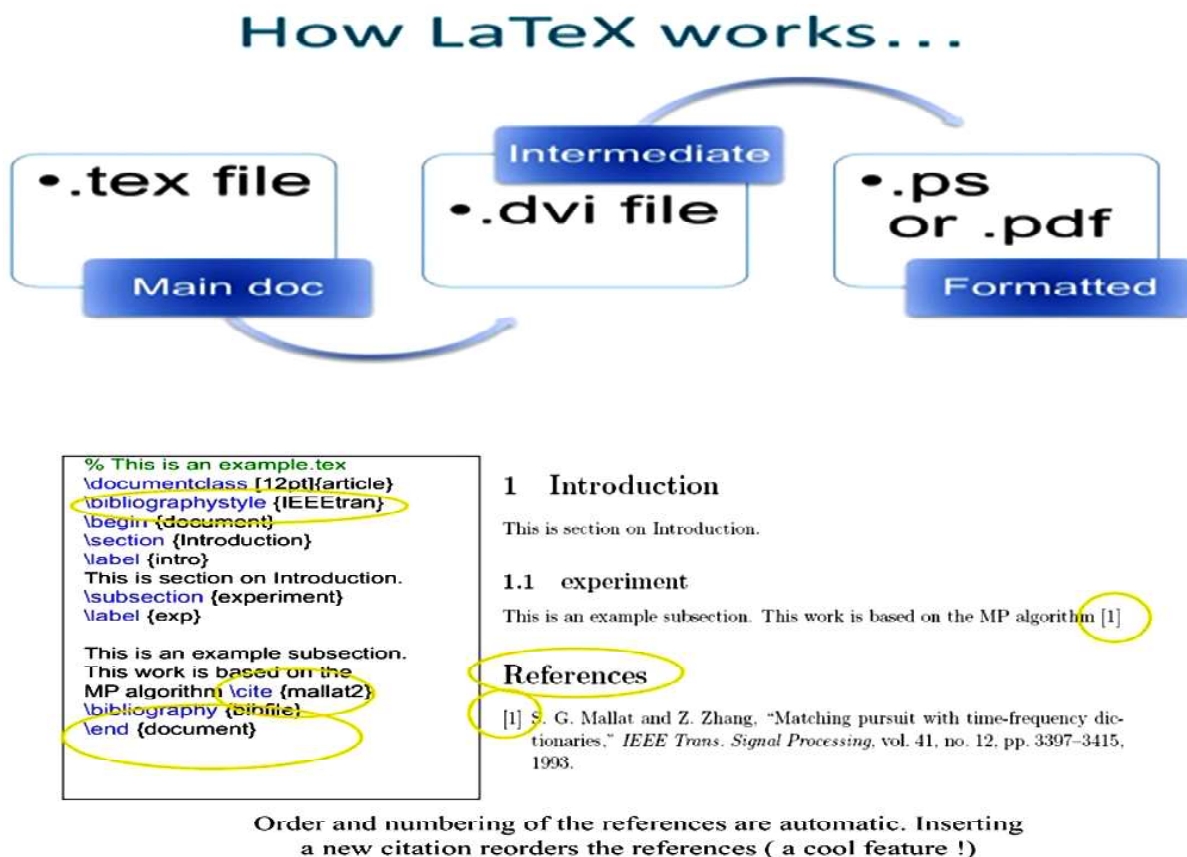
The participants of the workshop were shown the demonstration of the different features of LaTeX and also were trained on how to install and use this typesetting software for writing research articles.

3. Fuzzy Inventory Model

Ms. Tanzim S. Shaikh

Assistant Professor, Department of Statistics,
VPM's B. N. Bandodkar College of Science (Autonomous), Thane.

For undergraduate students who have just been introduced to the field of operations research and inventory control, the following will provide a basic understanding of the value and research of inventory control. Inventory control is a key aspect of supply chain management and is relevant to a wide range of industries, including manufacturing, retail, and logistics. Understanding inventory control can help students to prepare for careers in these industries as well as to research in this field. Inventory control is important for optimizing operations and maximizing profitability. Students can improve the efficiency and effectiveness of their operations by understanding the principles and practices of inventory management. It improves customer satisfaction by ensuring that the right products are available when and where they are needed. Consequently, stock shortages are minimized, and lost sales and reputation damages are avoided. Overall, effective inventory management is critical for optimizing operations and maximizing profitability for any business that holds inventory.

Writing Research paper is a key skill for academic success. In today's competitive academic environment, understanding how to conduct research, organize and present information, and write in a clear and concise manner is essential for success in higher education and beyond. Research paper writing helps students to develop communication skills. By writing research papers, students can learn how to express their ideas effectively and communicate their findings to others.

Fuzzy inventory model is a mathematical model that uses fuzzy logic to represent and solve inventory problems. It is used in situations where it is difficult to specify exact values or where there is a need to take into account the subjective judgment of decision makers.

In an inventory model, fuzzy logic can be used to represent the uncertainty that is inherent in forecasting demand, determining the optimal order quantity, or setting safety stock levels. Fuzzy logic can also be used to incorporate the preferences and objectives of decision makers into the inventory model. It can help decision makers to make more informed and effective inventory management decisions.

To write a research paper in fuzzy inventory model, begin by understanding the basics of fuzzy inventory models. This may involve reviewing relevant literature and getting a grasp of the key concepts and principles of fuzzy inventory management. Develop a clear research question or hypothesis related to fuzzy inventory models. This will guide your research and help you focus your efforts. Conduct research to gather information about fuzzy inventory models. Organize your research findings and analyze the data to draw conclusions. This may involve using statistical analysis or other methods to interpret the data and support your research question or hypothesis.

Write your research paper, using clear and concise language to present your findings and conclusions. Be sure to include a detailed discussion of the literature, a description of your research methods and data analysis, and a conclusion that addresses your research question or hypothesis. Review and revise your research paper as needed, paying attention to the organization, clarity, and coherence of your writing. Submit your research paper for review or publication, following the guidelines and requirements of the journal or conference where you are submitting it. By following these steps, you can write a well-organized and well-supported research paper on fuzzy inventory models or any other topic

POSTER PRESENTATION

1st Prize Winner of the Poster Presentation Competition

2nd Prize Winner
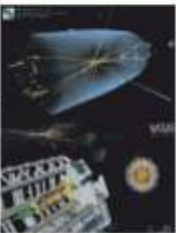
3rd Prize Winner
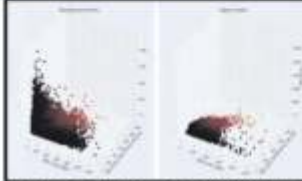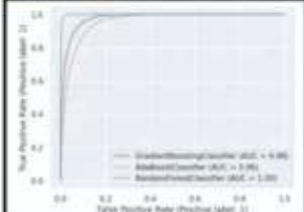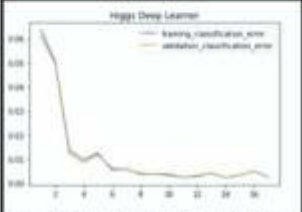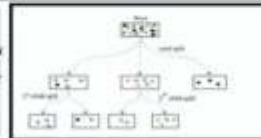
Glimpse of Poster Presentation

- Rohit Nair
(F.Y.B.Sc., St. Xavier's College, Mumbai)

72

- **Arpita Khoje, Vaishnavi Nagare**
(B. N. Bandodkar College of Science)

Applications of Fourier Series

-"Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them."

Joseph Fourier
(1768-1830)

What is Fourier series?

A Fourier series is a summation of harmonically related sinusoidal functions. also known as components or harmonics.

The result of the summation is a periodic function whose functional form is determined by the choices of cycle length (or period), the number of components, their amplitudes and phase parameters.

The number of components is theoretically infinite, in which case the other parameters can be chosen to cause the series to converge to almost any well behaved periodic function

(Pathological and Dirichlet–Jordan test)

$$f(x) = \frac{1}{2}a_0 + \Sigma_{n=1}^{\infty} a_n \cos nx + \Sigma_{n=1}^{\infty} b_n \sin nx$$

How it works?

Remember, the experiment we took in the school when we pass the beam of white light through the prism and we see different colors on the others side. That is what Fourier series does.

The Fourier series dissolves the complex periodic wave function in the sum of periodic trigonometric wave functions. It makes use of the orthogonality relationships of the sine and cosine functions.

Convergence of Fourier series means that as more and more components from the series are summed, each successive partial Fourier series sum will better approximate the function, and will equal the function with a potentially infinite number of components. The mathematical proofs for this may be collectively referred to as the Fourier Theorem.
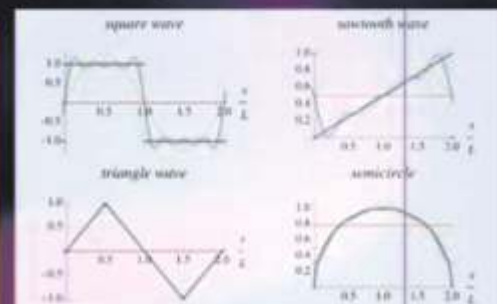
Where it is applied?

Signal Processing - It is used in many kinds of signal processing and considered as the best application of Fourier Series.It is a tool used for transforming a signal from time domain to the frequency domain. In the Fourier series the signal is decomposed into harmonically related sin functions.

Approximation Theory - We use Fourier series to write many functions as a trigonometric polynomial just like Taylor series. One problem of particular interest is that of approximating a function in a computer mathematical library, using operations that can be performed on the computer or calculator such that the result is as close to the actual function as possible. This is typically done with polynomial or rational approximations.

Control Theory - The Fourier series of functions in the differential equation often gives some predictions about the behavior's of the solution of the differential equation. They are useful to find out the dynamics of the solution.

Partial Differential equation - We use it to solve higher order partial differential equations of wave equation, heat eqaution and Schrodinger's wavefunctions. This method works for all infinite systems.

> **Did You Know !**
> It required 10 years to produce first proof of Fourier series after Fourier used in the heat equations

References:

1. https://www.youtube.com/watch?v=-qgreAUpPwM
2. https://math.stackexchange.com/questions/5794 53/real-world-application-of-fourier-series
3. https://en.wikipedia.org/wiki/Fourier_series
4. https://www.khanacademy.org/science/electrica l-engineering/ee-signals/ee-fourier-series/y/ee-fourier-series-intro

-CHAITALI G RANALKAR

- Chaitali Ranalkar
(B. N. Bandodkar College of Science)

Analysis of Starbucks using NPT and DataViz

INTRODUCTION

Starbucks was started in the year 1971. Their original business was roasting and retailing coffee, tea and spices. Currently, millions of customers are served in 33833 locations across 80 countries. It is a chain of stores that sell coffee, beverages as well as baked goods. In this study, author aims to test hypothesis regarding Starbucks factors in order to verify the relationship between these factors. Also, author wants to visualize the patterns of spending and frequency of visitation of customers on items they buy at Starbucks.

OBJECTIVE

1. To check is there a difference in Starbucks Price rate between regular customers and Nonregular customers.
2. Plotting the relationship between age group and how much time they spend at Starbucks by using grammar of graphics.

METHODOLOGY

This study is based on secondary data which has been collected from the website www.starbucks.data. This dataset contains only 122 customer's data.

In this study, relationship between Starbucks factors has been checked by using hypothesis testing. To checking whether the data is normal or non normal, I have use Shapiro Wilk test. For testing non parametric data, I make use of Wilcoxon signed-rank test to check the relationship between Starbucks factors. Data visualization is done here to visualize the data. To visualize the relationship between age groups of customers and their spending, I used grammar of graphics.

ANALYSIS AND INTERPRETATION

From Shapiro Test of both Variables, Price Rate & Loyalty(Regularity) of customers, We can see that the P-value is less than 0.05. The data is not normal, hence I performed the Wilcoxon signed-rank test to further analyze the data. Considering the following hypothesis as follows:

H_0: There is no significant difference in Starbucks price rate between regular and Non-regular customers.

H_1: There is significant difference in Starbucks price rate between regular and Non-regular customers.

Decision Criteria: Wilcoxon test shows that the P-value is again 8.053e-07 which is less than 0.05. Hence, we reject the NULL Hypothesis.

Conclusion: There is significant difference in Starbucks price rate between regular and Non-regular customers.
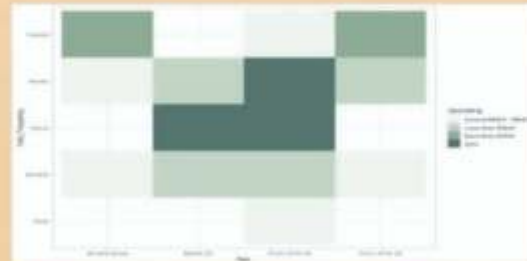
The visualization indicates visit frequency as per the age groups that visit the Starbucks store with respect to the amount, they spend in Starbucks for buying their items. We can see that ages above 30 spend more than RM40 and they visit weekly to the Starbucks store. we can also see that age is below 30 spend less than RM20 or zero amount towards the Starbucks buying and their visit frequency is rarely or never. almost all age groups visit monthly to the Starbucks store and spend somewhere Below RM40. people below age 30 who never visit Starbucks store spend 0 amount. Age is below 30 has the most visiting frequency than ages above 30.

CONCLUSION

1. Customer's Rating the price of Starbucks items depends on whether the customer is regular or not.
2. There isn't notable difference between the ambience and service rates at Starbucks.
3. The Starbucks management team can plan on improving their sales based on the visualization that shows visit frequency age and the spending of each customer.
4. The visualization is extremely helpful in understanding customer needs according to their visit frequency and the amount that they can spend as per their age.

LIMITATIONS

1. The dataset contains only 122 customer's data. This is very small for obtaining the accuracy of calculating results.
2. Study questions do not cover the whole data from dataset.
3. We don't know how the data was collected, there is a possibility for customer to submit answers in different formats or spellings. This would make the data collection fallacious.

REFERENCES

- https://www.starbucks.in/articles .
- https://en.wikipedia.org/wiki/Starbucks .
- https://www.geeksforgeeks.org/wilcoxon-signed-rank-test-in-r-programming/ .
- https://ggplot2.tidyverse.org/reference/geom_tile.html .

ACKNOWLEDGEMENTS

I would like to thank my teachers for their constant support. I am truly grateful and thankful to my sister who helps me improving my research paper.

MANDAR VASANT TARMALE
Vidya Prasarak Mandal's B. N. Bandodkar College of Science, Thane(W)

– Mandar Tarmale
3rd Winner
(B. N. Bandodkar College of Science)

Telecom Customer Churn Analysis

Monitoring customer churn is pivotal when it comes to keeping tabs on the subscription business. But is that all there is?
If company's churn rate is creeping up then what does it indicate ? How can the company form an action plan together to tackle it?
It's one thing to know the numbers, but if comapny does not know what they mean, company could leave it's customers along with the money on the table.
That's where customer churn analysis comes in!

Introduction

In the telecom sector, customers have a tendency to change operators very often, either to obtain better terms of use or to receive more advanced services that can meet customer expectations or to find a better customer experience. Analyzing customer turnover data can help the company understand the underlying reasons why customers are leaving the company. By implementing statistical analysis techniques and analysing the behaviour of customers in different segments, the company can discover why customers left and map out strategies to reduce churn. Customers with high a probability of switching can then be worked with to ensure they stay with their current provider.

Objectives

Telecom companies can learn more about their consumers by identifying the underlying factors behind the numbers using internal data. The main objectives of the analysis is to understand the customer profile for a customer that churned or retained and also to identify the major factors that appears to be the key drivers of customer churn. And best to map out the strategies for the company to retain the customers.

Data Collection and Research Methodology

Rollover data of telecommunications company provides telephone and Internet services to 7,043 customers. Each record represents a customer and includes details about their demographics, location, tenure, subscription services, status for the quarter (stayed or churned) and more. There were 37 columns in total.

Tools Used: MS Excel, Figma, Python, Tableau

Analysis and Findings

Customer Churn
■ No
■ Yes

? WHAT do we know about the customers that have churned?

Churning of customers by contract type

- Month-to-month
- One year
- Two year

Short term contract results in poor retention rate of customers.

Customer churn by offer

- None — 1,151 / 2,866
- Offer B — 414
- Offer E — 250 / 323
- Offer D — 827
- Offer A — 199
- Offer C — 234

Majority of customers who left had no offer.

Which Category is the biggest contributor in churning of the customer?

Competitor 45% | Dissatisfaction 17% | Attitude 17% | Price 11% | Other 10%

Competitors is the primary reason for the churn

Customers Distribution

☺ 5,174 Churned : No | 73.46%
Revenue: $15.09M | 74.35%

☹ 1,869 Churned : Yes | 26.54%
Revenue: $5.48M | 25.65%

Key Segments

7,043 **Total Customers**
$0.0138M **Total Refunds**
$21.371M **Total Revenue**

ANOVA Test for Difference in Numerical Features
This test is used to determine if there is significant differences between the means of Churn=YES and Churn=NO in numerical variables Tenure and Monthly Charges.
The alpha is set to 0.05.
If the p-value is lesser than alpha then we reject Ho

Ho: There is no statistically significant difference in means of the Churn groups in numerical variable monthly charge.

H1: There is statistically significant difference in means of the Churn groups in numerical variable monthly charge.
F_onewayResult (statistic=14.2429056834,
p-value=0.0001887361116)
p-value < 0.05 Hence Reject Ho

Ho: There is no statistically significant difference in means of the Churn groups in numerical variable Tenure.

H1: There is statistically significant difference in means of the Churn groups in numerical variable Tenure.
F_onewayResult(statistic=50.1687426,
p-value=7.82674709159946e-12)
p-value < 0.05 Hence Reject Ho

Chi-Square Test of Independence For Categorical Features: The goal of this analysis is to investigate the relationship of the categorical features to the dependent/target variable Churn=YES. The test will determine if there is significant difference in the proportion of Churns between the groups in the categorical variable. The **alpha is set to 0.05.** If the **p-value is less than alpha then reject the null hypothesis** and conclude that the proportions of Churn is dependent with the group in the categorical variable.

Chi-Square Test for Internet Service and customer churn	Chi-Square Test for Online Security and Churn	Chi-Square Test for Phone Service and Churn	Chi-Square Test for Premium-Tech Support & churn
Ho : Customer churn and Internet Service are independent. H1: Customer churn and Internet Service are not independent	Ho: Customer churn and Online Security are independent H1: Customer churn and Online Security are not independent	Ho: Customer churn and Phone services are independent. H1: Customer churn and Phone services are not independent.	Ho: Customer churn and Premium-tech support are independent. H1: Customer churn and Premium-tech support are not independent.
degrees of freedom = 2	degrees of freedom = 2	degrees of freedom = 1	degrees of freedom = 2
Results: probability = 0.950, critical = 5.991, stat = 728.696	Results: probability = 0.950, critical = 5.991, stat = 846.677	Results: probability = 0.950, critical = 3.841, stat = 0.874	Results: probability = 0.950, critical = 5.991, stat = 824.926
Decision based on Chi-Square Statistics: The Churn are dependent on Internet Service (Reject Ho)	Decision based on Chi-Square Statistics: The Churn are dependent on OnlineSecurity : (Reject Ho)	Decision based on Chi-Square Statistics: The Churn are independent on PhoneService (Fail to reject Ho)	Decision based on Chi-Square Statistics: The Churn are dependent on TechSupport : (Reject Ho)
Decision based on p-value: alpha = 0.05, p = 0.0000000 The Churn are dependent on Internet Service(Reject Ho)	Decision based on p-value: alpha = 0.05, p = 0.000 The Churn are dependent on Online Security : (Reject Ho)	Decision based on p-value: alpha = 0.05, p = 0.350 The Churn are independent on PhoneService (Fail to reject Ho)	Decision based on p-value: alpha = 0.05, p = 0.002 The Churn are dependent on TechSupport : (Reject Ho)

Inferences and Reccomendations

1. Customers with **Monthly Contracts** are a high Churn Risk. Identified Churn Risks should be handled on priority.

2. Due to superior products or services offered by competitors, 45% of customers churned. The company should perform a competitive analysis and benchmark its products and services against those of its competitors to better position itself in the marketplace. By doing this, the business can anticipate a reduction in the rate of churn which will improve customer retention.

3. 17% of customers were dissatisfied due to attitude of support, which can lead to word of mouth to spread from bad interactions.

4. Since any offer is better than no offer, company must use offers to incentivize customers to refer the services to others and get on one/two year contracts.

5. ANOVA test implies that the change in the value of the numerical variables has effect on whether customer will stay subscribed or not.

6. Chi-Square test implies that the factors such as Internet service, online security and technical support have a significant impact on customer attrition.

References: http://bit.ly/123 http://bit.ly/x1 http://bit.ly/xtst https://guest.tableau.com/profile/shreya... https://images.jeg.org/banner/014x0

Author: Shreya Nadkarni , VPM's B.N.Bandodkar College of Science (Autonomous) Thane

- Shreya Nadkarni
(B. N. Bandodkar College of Science)

- Shraddha Dwivedi
(B. N. Bandodkar College of Science)

– Sarika Jadhav
(B. N. Bandodkar College of Science)

GOVERNMENT OR PRIVATE JOB?

To analyse the preference of employment type among students

HYPOTHESIS

NULL HYPOTHESIS (HO)

1. CHOICE OF EMPLOYMENT DOESN'T DEPENDS ON THE GENDER OF THE STUDENTS.
2. PREFERENCE OF EMPLOYMENT DOESN'T DEPENDS ON THE AGE OF THE STUDENTS

ALTERNATE HYPOTHESIS (H1)

1. CHOICE OF EMPLOYMENT DEPENDS ON THE GENDER OF THE STUDENTS.
2. PREFERENCE OF EMPLOYMENT DEPENDS ON THE AGE OF THE STUDENTS.

Employment preference among students

- Any job
- Merchant Navy
- Family Business
- Self Business / Startup
- Government job
- Private Job

■ Private Job　■ Government job　■ Self Business / Startup

DO YOU KNOW ?

Out of **22 crore** applicants, only **7.2 Lakh** got govt jobs in 2014-22

3.4% workforce of India works in public sector (ILO-2014)
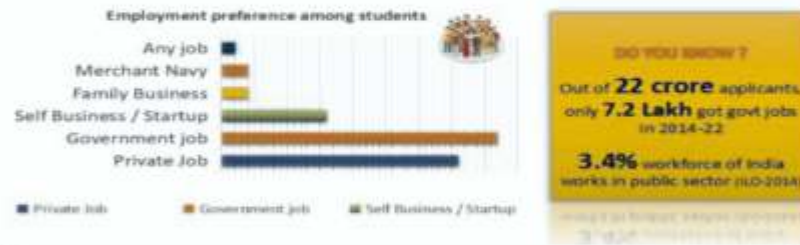
METHOD

The **Pearson's Chi-Square** statistical hypothesis is a test for independence between categorical variables.

The objective of the test is to conclude whether the two variables are related to each other.

Hypothesis:
We start by defining the **null hypothesis (HO)** which states that there is **no relation** between the variables. An **alternate hypothesis (H1)** would state that there is a significant **relation** between the two.

Using p-value:
We define a significance factor to determine whether the relation between the variables is of considerable significance. Generally a significance factor or alpha value of 0.05 is chosen. This alpha value denotes the probability of erroneously rejecting HO when it is true. A lower alpha value is chosen in cases where we expect more precision. If the p-value for the test comes out to be strictly greater than the alpha value, then HO holds true.

CHI SQUARE TEST RESULT

$X^2(4 \mid N=106) = 11.3107, p = 0.010159$

The chi-square statistic is 11.3107. The p-value is 0.010159. The result is significant at $p < .05$.

$$p < \alpha$$

Since p value of 0.010159 is smaller than significance level which we have considered as $\alpha=0.05$. So we reject null hypothesis HO and accept the alternate hypothesis H1 which states that there is a significant relation between the two variables

Thus, choice of employment is dependent of the gender of the student.

RESULT & CONCLUSIONS

1. From **Chart 1: EMPLOYMENT PREFERENCE AMONG STUDENTS** it can be seen that Government Sector Job has the highest preference among students followed by Private Jobs, Business & Startups.
2. From **Chart 2.1: PREFERENCE AMONG FEMALE** we can see that female students prefer Government Job followed by Private Job and Business & Startups, while from **Chart 2.2: PREFERENCE AMONG MALE** we can see that male students give higher preference to Private Sector Job, followed by Private Jobs and Business & startups. As observed **from Chart 2.1 and 2.2** the choice of employments depends on the gender. Thus **our first null hypothesis is failed and is rejected.**
3. As observed from **Chart 3.1** the students of age group 15 to 20 years prefer Business & Startups while from chart 3.2 the students of age group prefer Government Job. Thus our **second null hypothesis is failed and is rejected.**

- Manthan Mohite
(B. N. Bandodkar College of Science)

India's Consumer Automobile Market:
Big, Bigger, Biggest

Rajeev Ghade, TYBSC Statistics, B.N Bandodkar College of Science

Introduction

- The automobile industry for any country has been a good indicator of how well the economy is doing, as the automobile sector plays a key role in both macroeconomic expansion and technological advancement.
- The automobile industry in India is currently the **fourth-largest** in the world, worth more than **$100 billion**, which contributes 8% to India's total exports and accounts for approximately 2.5% of India's GDP.

2 Wheelers and 4 Wheelers Sales

- And now that we have entered into India's *Azaadi ka Amritkaal* we are here to witness some fast-paced growth. It is predicted that India will have the largest young and middle-class population in the whole world, which will drive consumption in our economy.
- If everything goes as per the plan, our GDP per capita is bound to grow from the current **$2470** to a minimum of **$10,000** which will place us with upper-middle-income economies but taking into consideration of our denominator factor of GDP per capita i.e. population we will easily have an economy of more than **$15 trillion**.
- Now if all this economic growth gets to factor into the automobile industry then surely we are here to witness a behemoth in making.

Methodology

- Acquisition of Data from SIAM Annual Reports and World Bank
- Data Entry and processing for further programming in R
- Creating a multiple linear regression model for 2 Wheelers sales where the Dependent Variable: Sales of 2 Wheelers Independent Variables: Gini Coefficient, Unemployment %, Gross Domestic Savings % of GDP
- Creating a weighted multiple linear regression model for 4 Wheelers sales where the Dependent Variable : Sales of 4 Wheelers Independent Variables: Gini Coefficient, Unemployment %, Gross Domestic Saving % of GDP
- To fit the model variable transformation is done of Gini coefficient.
- For future prediction extrapolation is done by multiplying the output value by the factor of GDP per capita present right now.

Assumptions

- To predict the future sales value we will assume the Gini Coefficient to be 37%.
- Unemployment Rate to be at a mean of 27 years i.e. 5.7%.
- Gross Domestic Savings to be at a mean of 27 years i.e. 30%.
- If there are any changes in future we can change the values in the model to get the predicted sales number.

Analysis

Multiple Linear Regression Model

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon$$

2 Wheelers Fitted Model

2 Wheelers Sales = 249709635 - 30950774(Ginicoeff) - 384444463(Unemployment) - 50037308(savings)

4 Wheelers Fitted Model

4 Wheelers Sales = 47908023 - 63967381(Ginicoeff) - 23948653(Unemployment) - 9656770(savings)

Results & Conclusions

Future Prediction of Sales V/S change in GDP per capita ($)

- From the 2 Wheelers Sales Model we can predict that when our country's GDP per capita would be $10,000 then sales would be **7.20 crore**
- From the 4 Wheelers Sales Model we can predict that when our country's GDP per capita would be $10,000 then sales would be **1.36 crore**
- From the model we can also conclude that Sales are directly proportional to Gini Coefficient and inversely proportional to Unemployment and Savings rate

Bibliography

https://www.siam.in/cpage.aspx?mpgid=42&pgidtrail=89
https://data.worldbank.org/indicator/SL.UEM.TOTL.ZS?locations=IN
https://data.worldbank.org/indicator/NY.GDP.PCAP.CD?locations=IN
https://data.worldbank.org/indicator/NY.GDS.TOTL.ZS?locations=IN

– Rajeev Ghade
(B. N. Bandodkar College of Science)

Comparison of Electric Vehicles and Fuel Vehicles In India

- **Gayatryi More, Laxmi Nishad**
(B. N. Bandodkar College of Science)

- Rohan Patil, Siddhesh Chaugule, Divya Singh
1st Winner
(B. N. Bandodkar College of Science)

- Jui Shinde, Hemangi Posti, Anjali Pal
2nd Winner
(Dr. V.N. Bedekar Institute of Management Studies)

Fidelity and RFC Knowledge of Ethnoveterinary Medicinal Plants used by Shahapur District, Thane Residents

Data Analysis:

The obtained data is presented in tables. Simple descriptive statistics were used to analyse them.

Informants Selected for Sampling

During a survey, 96 respondents (57 men and 39 women) were chosen for interviews. During the interview, informants' ages and educational backgrounds were recorded.

Analysis of ethnoveterinary data in a quantitative manner RFC

Relative Frequency of Citation

The purpose of the RFC is to develop a priority order among the listed species, and its value is determined by the number of participants who have mentioned a specific species as a medicinal plant or good feed for livestock, indicating its significance. The RFC was calculated using the following equation: RFC = FC/N. Friedman et al. (1986).

Where FC denotes the number of of participants who identified a specific plant species as an excellent medicinal plant and N denotes the total number of participants included in the study.

Fidelity Level

The fidelity level (FL), which is the percentage of informants who claim to have used a specific plant for the same major purpose, was calculated for the following diseases or ailments:

$FL (\%) = (Np / N) \times 100$

Where Np is the number of informants who claim to have used a plant species to treat a particular disease, (Friedman et al., Yaniv, Dafni, & Plaewitch, 1986 When all participants refer to a plant for the same purpose, the plant receives the highest FL value (100%).

Use Value

The use value is an ethnobotanical key that is used to assess the relative value of useful plant species.

$UV = \Sigma Ui / n$

where Ui is the total number of informations and n is the number of usage reports cited by each informant for a given plant.

References

1. Friedman, J., Yaniv, Z., Dafni, A., and Palewitch, D. (1986). A preliminary classification of the healing potential of medicinal plants, based on a rational analysis of an ethnopharmacological field survey among Bedouins in the Negev Desert, Israel. Journal of Ethnopharmacology, ; 16(2-3) ; 275–doi:10.1016/0378-8741(86)90094-2, PMID:pubMed: 3747566.

2. Yabesh, J. E., Prabhu, S., and , Vijayakumar, S. (2014). An ethnobotanical study of medicinal plants used by traditional healers in silent valley of Kerala, India. Journal of Ethnopharmacology, 154(3), , 774–789, doi: 10.1016/j.jep.2014.05.004.

3. Albuquerque, U.P.; Lucena, R.F.; Monteiro, J.M.; Florentino, A.T.; Cecilia de Fatima, C.B.R. Evaluating Two Quantitative Ethnobotanical Techniques. Ethnobot. Res. Appl. 2006, 4, 51-60. (CrossRef)

Table 1: Demographic characteristics of respondents of Livestock owners

Table No.1. List of Medicinal Plants used in ethnoveterinary

– Heena Mapkar, Amala Karulkar, Siddhi Mhatre
(B. N. Bandodkar College of Science)

Knowledge of Ethnoveterinary Medicinal Plants Used by Shahapur District, Thane Residents

Abstract:

A field research extensive survey was undertaken during the years 2021-2022. This kind of to learn about medicinal plants advantages The Palghar district of this study has provided the use of 41 plant species under 44 genera belonging to 33 families. Plant species, local names, families' usable parts, and ethnoveterinary remedies are listed alphabetically. The members of Fabaceae, Asteraceae, Euphorbiaceae and Zingiberaceae lead with three each, followed by Annonaceae, Lamiaceae, Moraceae, Solanaceae and Amaranthaceae with two each and the remaining with one each. The traditional medicinal practitioners and the local people of the Shahapur Dist. Thane area of Maharashtra used plants to cure diarrhea, dysentery arthritis, eye damage, skin diseases, milk problem, wounds, bone fractures, fever, and other illnesses in animals.

Introduction

The process of treating various ailments in animals and maintaining their health using traditional knowledge, skills, and practices is known as ethno-veterinary medicine. The term "ethno-veterinary" Dr. Constance M. McCorkle gave an introduction in 1986. Her article "An Introduction to Ethnoveterinary Research and Development" is where it was first used.

There are just a few documented articles on ethnoveterinary uses of medicinal plants in different regions of India (Gaur et al. 2010, Tiwari and Pandey 2010; Ramana 2008, for instance). Maharashtra has produced very few publications as well (Patil, et.al. 2001). Wath and Jambu conducted an ethnoveterinary study in the Amravati district's Melghat area (2010).

Materials and Methods

Location of the study area

Thane district, which makes up 3.11 % of Maharashtra's total land area, is located between 18°42' N and 20°20' N latitudes and 72°4' E and 73°48' E longitudes.

Survey

From 2021 to 2022, 10 survey visits to 25 villages in the Thane area were scheduled.

Identification of Plants

Specimens were identified using local floras and supporting material for ethnomedical purposes as published in various works of literature, books etc. (Cook, 1908, Almeda, 2003, Naik, 2004.)

Results and Discussion

Many herbs are reportedly used to treat milking, diarrhoea, arthritis, eye damage, milk difficulty, wounds, bone fractures, fever, and other problems of various animals, according to surveys and studies based on interactions with traditional healers and local people of the Shahapur,Thane district. The observed herbs for use in veterinary medicine are discussed in conversations with respondents, along with their observations of conventional medical procedures and the outcomes.

Conclusion

1. Order people have more experience in traditional knowledge.
2. The doses of medicine also vary according to animal and its weight.
3. Various plants from different families have been effectively used for treating ailments in animals.
4. Our findings present considerable potential for further scientific research on these plant species, which can lead to development of cheaper & more efficacious drugs.
5. To increase scope of plant based medicines which may prove more effective & safe than synthetic medicine.

References

1. Almeda M. R. (2003). Flora of Maharashtra Orient Press, Mumbai.
2. Cooke T. (1908). The Flora of the presidency of Bombay. Vol. I, II, III. Botanical Survey of India Calcutta Rep. Edn.1967.
3. McCorkle CM (1986) An introduction to Ethnoveterinary Research and Development. J. Ethnobiology 6:129–149.
4. Naik V. N. (2004). Identification of Common Indian Medicinal Plants. Scientific Publishing, Jodhpur.

Table No.1. List of Medicinal Plants used in ethnoveterinary

Sr. No.	Botanical Name	Local Name	Family	Ethnoveterinary Uses

— Heena Mapkar, Amala Karulkar, Siddhi Mhatre
(B. N. Bandodkar College of Science)

TRADITIONAL HERBAL COSMETICS USED BY TRIBAL WOMEN IN THE PANVEL RAIGAD DISTRICT

Category: C3   **Slot No.**   **Level: UG**

ABSTRACT

A field research extensive survey was undertaken during the years 2020-2021. This kind of to learn about medicinal plants' advantages. The Raigad district of this study has provided the use of 28 plant species under 27 genera belonging to 24 families. The botanical names of plant species and their regional names, the useful components of plant families, and traditional cosmetics uses are presented. The members of Rutaceae, Euphorbiaceae, and Zingiberaceae lead with two each, and the remaining with one each. *Curcuma longa*, *Sapindus mukorossi* Gaertn., *Lawsonia inermis* L., *Ocimum sanctum*, *Citrus limon* (Linn.) Burm.f., *Sesamum indicum*, *Ficus benghalensis* L., and *Cicer arietinum* flour are the most frequently recorded plants. The current study identified 28 plants used by tribal women in Panvel, Raigad district for skin and hair care. Twelve of them were for skin care, while sixteen were for hair care. Plants have been used for the human beautification in ancient times, and various types of herbs have been used to maintain beauty.

Keywords: cosmetics, indigenous, ethnomedicine, herbal, ancient

INTRODUCTION

The word cosmetic comes from the Greek word kosmtikos means the ability to decorate. Cosmetics that can be defined as rubbing, pouring, sprinkling, or pouring spraying, or applying to the human body or parts thereof Purify and enhance beauty. Cosmetic Medicines are collectively called cosmaceuticals. Numerous medicinal plants in Vartya kadhya were mentioned in Ayurvedic literature, particularly Charak Sahita. Herbs such as Chandan, Haldi,etc. are used to enhance a glowing complexion. Individuals' skin and hair beauty is determined by their health, habits, routine, job, climate, conditions, and maintenance (Sharma, 1998).

Traditional Women herbal formulation

MATERIALS AND METHODS

Selection of the Site

Panvel is a city in Maharashtra's Raigad district, close to Navi Mumbai. Panvel is a city in the Raigad district of Maharashtra. Panvel is also known as the Raigad Gate since it is the first city encountered when approaching Raigad from the east. It is also one of the district's most populated and developed cities. The city of Panvel is located on the banks of the Gadhi River, which drains into the Arabian Sea. On two sides, it is likewise encircled by hills. Throughout the year, the weather is sunny. During the monsoon, there is significant rainfall. May is the hottest of the year. The average temperature in May is 34.3 °C. In January, the lowest average temperature of the year are about 23.4 °C. More days in the summer, from mid-March-May, the temperature hovers around 40 °C. The peak temperature was approximately 44-45 °C. The Panvel has an average yearly temperature of 27.6 °C.

Identification of plants

Specimens were identified using local floras and supporting material for ethnomedical purposes as published in various works of literature, and books. (Almeda, 2003; Cook, 1908; Isinak Rao, 1977;Imand Srivastava, 1989 Naik, 2004; Shah, 1978;). Specimens were dried and prepared as herbarium specimens or supporting documentation and stored permanently at the herbarium of the Department of Botany. VP M's B N Bandodkar College of Science, Thane Maharashtra.

Survey

From 2020 to 2021, 10 survey visits to 21 villages in the Panvel Dist. Raigad area were scheduled. The knowledge was obtained from village elders, traditional healers and practitioners, experienced and old farmers, and rural residents who were informed. For ethnobotanical and cosmetics information, they were questioned.

OBSERVATION

They had to answer the survey questions in Marathi. The information was cross-verified with appropriate and specialized use of plants and plant components. The questions often concern the sorts of ailments, treatment methods, rates of cure, periods or duration of the treatments, etc.

According to surveys and studies based on interactions with traditional healers and local people of the Panvel Dist Raigad district. The observed herbs for use in herbal cosmetics are discussed in conversations with respondents, along with their observations of conventional medical procedures and the outcomes.

The present study shows that the local people use a significant traditional knowledge of personal hygiene. The main objective of the current research study was to investigate and gather important information on the ethno-medicinal herbs used in cosmetics.

The current study identified 28 plants used by tribal women in Panvel, Raigad district for skin and hair care. Twelve of them were for skin care, while sixteen were for hair care. Plants have been used for human beautification since ancient times, and various types of herbs have been used to maintain one's beauty. According to the study, tribal communities use plants as herbal cosmetics as part of their ethnic culture, but they have also used plants in their regular healthcare practices since ancient times.

Curcuma longa, *Sapindus mukorossi* Gaertn., *Lawsonia inermis* L., *Ocimum sanctum*, *Citrus limon* (Linn.) Burm.f., *Sesamum indicum*, *Ficus benghalensis* L., and *Cicer arietinum* flour are the most frequently recorded plants.

Plants used in cosmetics not only improve beauty, but they also have medicinal properties, so using medicinal plants in cosmetics may provide some recommended medicinal advantages for the skin and body.

RESULT AND DISCUSSION

Table 1 below lists the ethnobotanical and cosmetic medicinal plants that were identified through field Research. 28 plant species from 24 families and 27 genera were identified, classified, and discussed. The botanical names of plant species and their regional names, the useful components of plant families, and traditional cosmetics uses are presented. The members of Rutaceae, Euphorbiaceae, and Zingiberaceae lead with two each, and the remaining with one each.

Plants used in cosmetics not only improve beauty, but they also have medicinal properties, so using medicinal plants in cosmetics may provide some recommended medicinal advantages for the skin and body. Natural cosmetics are becoming more popular than synthetic cosmetics. More plants must be scientifically screened for their cosmetic potential. Flavonoids, polyphenols, terpenoids, antioxidants, carotenoids, and vitamins have been identified as being responsible for the cosmetic effects of various medicinal plants. Most selective research is required to prove the exact relationship between chemical constituents and the cosmetic effect of the selected plant, which may aid in the future use of the selected chemical compound in cosmetic products.

The findings of recent studies supported the findings of the previous researchers from Ashwin et al (2009), Raijwar (CG.); Panvel (2017); Jaipur Yadav (2017) Rajasthan, Mohd 2021 from Uttar Pradesh, Kokkar et al. Kolhapur Maharashtra.

The results of the current study indicate that there is still a lot of information that needs additional study. The study of plant species information becomes less economical as it is more successful at treating ailments because it is indigenous and is beneficial local people.

Table 1: Cosmetic Medicinal plants used by tribal women

| No.	Botanical name	Local name	Family	Cosmetic uses

CONCLUSION

The documentation of this information is important for the communities and future generations, as well as for scientific evaluation of traditional knowledge's value applications in herbal cosmetics. Traditional medicinal plant remedies are favoured by the local people due to their inexpensive cost and lack of negative effects. The study's findings can thus be used to inform the public to protect the region's natural wealth of bio-resources before they fall prey to large-scale deforestation activities, which are underway in Raigad district. The information gathered shows that not only are many preparations made from a single plant but a mixture of several plants is easily used. Most of the preparation is consumed orally and applied topically. Many people in the studied area still believe in herbal remedies, which play an important role in their communities' lives.

REFERENCES

1. Almeda, M. R. (2003). Flora of Maharashtra orient press, Mumbai.
2. Aparna, P (2017). Studies on traditional herbal cosmetics used by tribal women in district Jaipur (Rajasthan) Journal of Physiological Research, 30(1), 23-30.
3. Ashavni, M. S., Banubhai, M., Saraf, S., & Saraf, S. (2008). Herbal cosmetics: "Trends in skin care formulations Pharag. Rev., 3(5) pp. 82-89.
4. Atif, Mohd., Yadav, P., Maus, M., Khan, S., Ahemad, S. (2021). Herbal cosmaceuticals International Journal of Creative Research Thoughts (IJCRT), 9(1), 3674-3681.
5. Cooke, T. (1967). The Flora of the of Bombay, I, II, III. Botanical of India, Calcutta.
6. Gonzalez-Minero, F. J., Bravo-Diaz, L., & Ayala-Gomez, A. (2020). Rosmarinus officinalis L.(): An Ancient plant with uses in personal healthcare and cosmetics, Cosmetics, 7(4), 77.https://doi.org/10.3390/cosmetics7040077.
7. Jain, S. K., Rao, R. R. (1977) A handbook of field and herbarium methods. Today and Tomorrow Printer Publisher

- Sadiya Jakati, Kahekasha Shaikh
(B. N. Bandodkar College of Science)
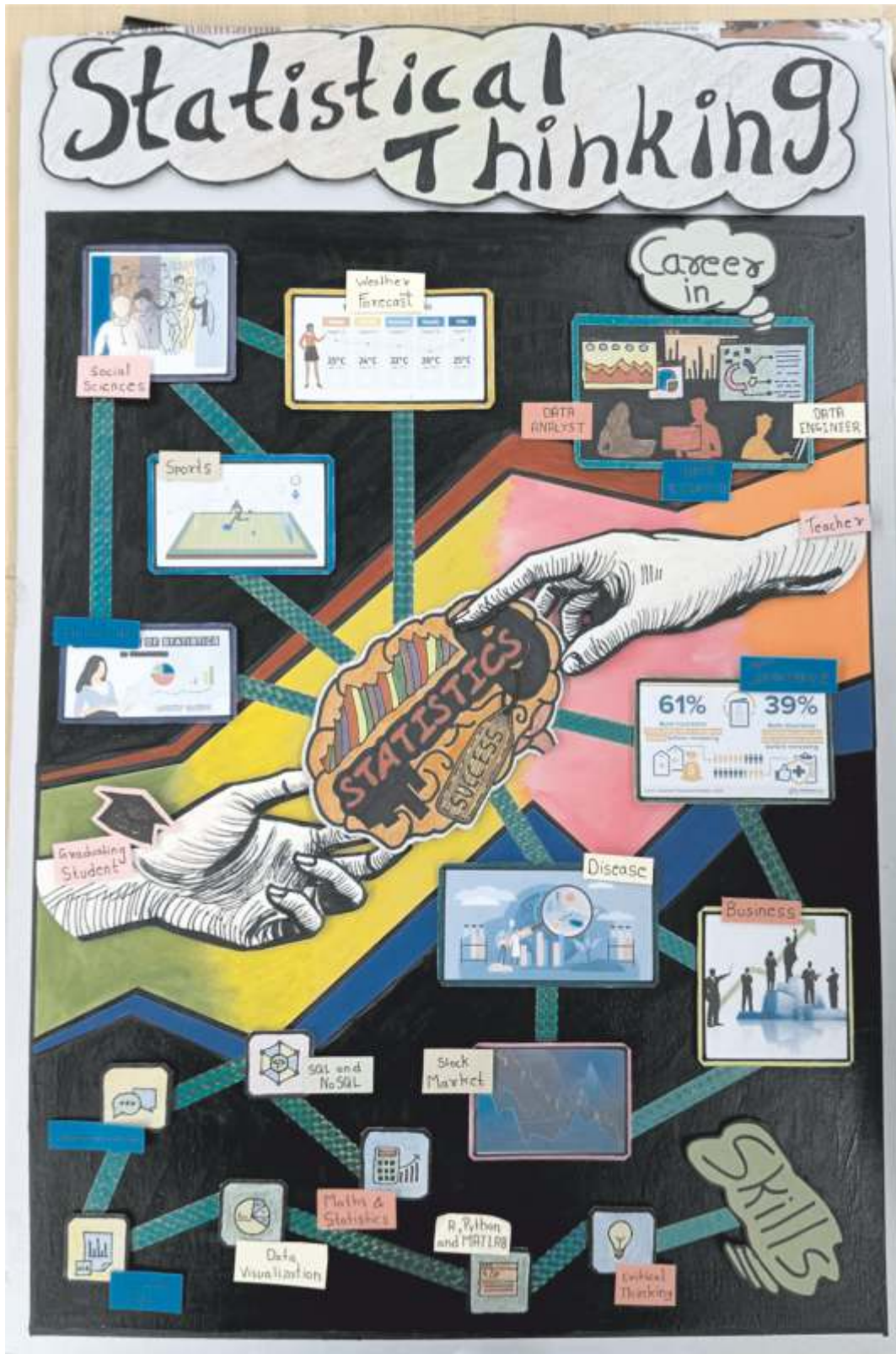
- Ajayraj Pasi
(R. J. College)

NCAMS - 2023